The error of Lagrange interpolation is

$$E_n(x;f) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)\dots(x-x_n)$$

where ξ in (x_0, x_n).

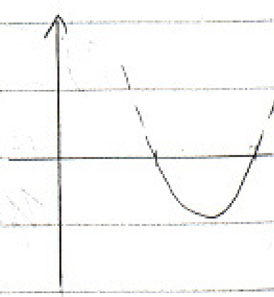
Ex. The function $f(x) = \sqrt{x}$ has been linearly interpolated by a degree 1 polynomial at the points 1, 2. Find the smallest upper bound for the absolute value of the error $|E_1(x;f)|$.

$$E_1(x;f) = \frac{f''(\xi)}{2!}(x-1)(x-2)$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} \qquad f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} = -\frac{1}{4x^{\frac{3}{2}}}$$

$$|f''(x)| = \frac{1}{4x^{\frac{3}{2}}} \rightarrow \text{decreasing}$$

$$|f''(x)| \le \frac{1}{4 \cdot 1^{\frac{3}{2}}} = \frac{1}{4}$$



$$|(x-1)(x-2)| \le \frac{1}{4}$$

$$|E_1(x;f)| \le \frac{\frac{1}{4}}{2!} \cdot \frac{1}{4} = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{32}$$