3.3 Hermite Interpolation

1) Osculating Polynomials

So far we constructed polynomials which agree with the values of the function in $n+1$. Hermite's interpolation is about constructing a polynomial which agrees with the function and its derivative.

<u>Problem:</u> Given $n+1$ distinct points

$$x_0, \ldots, x_n$$

and given nonnegative integers

$$m_0, \ldots, m_n$$

with

$$m = \max\{m_0, \ldots, m_n\}$$

Assume $f \in C^m[a,b]$.

Find a polynomial of least degree which agrees with the function and all its derivatives of order $\leq m_i$ at x_i, that is

$$P^{(k)}(x_i) = f^{(k)}(x_i) \qquad \begin{matrix} k = 0, \ldots, m_i \\ i = 0, \ldots, n \end{matrix}$$

The degree of the polynomial is at most equal to the number of conditions it has to satisfy minus one.