

4) Convergence of the Bisection method. Number of iterations

Theorem 2.1 Suppose $f \in C[a, b]$ and $f(a)f(b) < 0$.
The Bisection method generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of f with

$$\begin{matrix} (*) \\ (*) \end{matrix} \quad |p_n - p| \leq \frac{b-a}{2^n} \quad \text{with } n \geq 1$$

Proof:

$$|p_n - p| \leq \frac{b_n - a_n}{2} = \frac{b-a}{2^{n-1}} \cdot \frac{1}{2} = \frac{b-a}{2^n}$$

- Consequently the sequence $\{p_n\}_{n=1}^{\infty}$

converges to p with rate of convergence $O\left(\frac{1}{2^n}\right)$. Thus

$$p_n = p + O\left(\frac{1}{2^n}\right)$$

Q.E.D

Note: $\begin{matrix} (*) \\ (*) \end{matrix}$ gives only an upper bound for the error. The actual error could be much smaller.