

2) The sequence  $p_n = 10^{-2^n}$  converges to zero quadratically.

$$\frac{|p_{n+1} - 0|}{|p_n - 0|^\alpha} = \frac{10^{-2^{n+1}}}{|10^{-2^n}|^\alpha} = \frac{10^{-2^{n+1}}}{10^{-2^n \cdot \alpha}} \rightarrow 1 \text{ if } \alpha = 2$$

Note: Quadratically convergent sequence converge much faster

2) Convergence of fixed point iteration

Consider a sequence generated by

$$p_{n+1} = g(p_n)$$

Theorem 2.7. Let

a)  $g \in C[a, b]$

b)  $a \leq g(x) \leq b$

c)  $\exists 0 < k < 1$  with  $|g'(x)| \leq k \forall x \in (a, b)$

If  $\{g'(p) \neq 0\}$ , then  $\forall p_0 \in [a, b]$   
the sequence  $\{p_n\}_{n=1}^{\infty}$  converges linearly  
to the unique fixed point  $p$ .