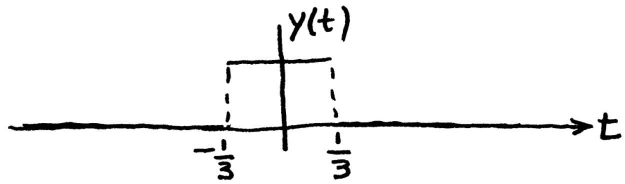
PROBLEM 11.1:

$$x(t) = u(t+1) - u(t-1) = \begin{cases} 1 & -1 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$y(t) = x(3t)$ is narrower; its duration $= \frac{2}{3}$ sec.



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PROBLEM 11.2:

(a) The Fourier Transform (FT) of $\delta(t)$ is 1. Thus, the FT of $\delta(t-t_d)$ is $1e^{-j\omega t_d}$

using linearity of the FT.

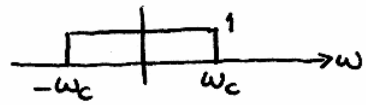
$$FT\{x(t)\} = FT\{\delta(t+1)\} + FT\{2\delta(t)\} + FT\{\delta(t-1)\}$$

$$X(j\omega) = e^{j\omega} + 2 + e^{-j\omega}$$

$$= 2 + 2\cos\omega$$ if you simplify

(b) $\dfrac{\sin(100\pi(t-2))}{\pi(t-2)}$ is a shifted "sinc" function

The FT of $\dfrac{\sin(\omega_c t)}{\pi t}$ is a rectangle



This F.T. can be found in Table 12.1.

It can also be written in terms of unit steps as $u(\omega+\omega_c) - u(\omega-\omega_c)$. In this case, $\omega_c = 100\pi$ rad/s

Using the shift property with $t_d = 2$

$$X(j\omega) = \{u(\omega+100\pi) - u(\omega-100\pi)\} e^{-j2\omega}$$

(c) The F.T. of $e^{-at}u(t)$ is $\dfrac{1}{a+j\omega}$.

$$x(t) = e^{-t}u(t) - \underbrace{e^{-t}u(t-4)}_{\text{This is NOT a pure shift}} = e^{-t}u(t) - e^{-4}\underbrace{e^{-(t-4)}u(t-4)}_{\text{This re-write shows the shift of both terms}}$$

$$X(j\omega) = \frac{1}{1+j\omega} - e^{-4}\frac{e^{-j4\omega}}{1+j\omega}$$

$$= \frac{1 - e^{-4(1+j\omega)}}{1+j\omega}$$

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PROBLEM 11.3:

The general approach is to use Tables plus some algebraic manipulations:

(a) $\dfrac{j\omega}{0.1+j\omega}e^{-j0.2\omega} = X_1(j\omega)e^{-j0.2\omega}$ ← use time shifting

If $X_1(j\omega)=\dfrac{j\omega}{0.1+j\omega}$, then $X_1(j\omega)=j\omega X_2(j\omega)$ ← use derivative

If $X_2(j\omega)=\dfrac{1}{0.1+j\omega} \Rightarrow x_2(t)=e^{-0.1t}u(t)$

$\Rightarrow x_1(t)=\dfrac{d}{dt}x_2(t)=e^{-0.1t}\delta(t)-0.1e^{-0.1t}u(t)=\delta(t)-0.1e^{-0.1t}u(t)$

$x(t)=x_1(t-0.2)=\delta(t-0.2)-0.1e^{-0.1(t-0.2)}u(t-0.2)$

(b) $X(j\omega)=2+2\cos\omega=2+e^{-j\omega}+e^{j\omega}$ ← use shifting

$x(t)=2\delta(t)+\delta(t-1)+\delta(t+1)$

(c) use Table entry $\dfrac{1}{a+j\omega}\longrightarrow e^{-at}u(t)$

$x(t)=e^{-t}u(t)-e^{-2t}u(t)$

(d) use Table entry: $2\pi\delta(\omega-\omega_0)\longrightarrow e^{j\omega_0 t}$

$X(j\omega)=j\frac{2\pi}{2\pi}\delta(\omega-100\pi)-j\frac{2\pi}{2\pi}\delta(\omega-(-100\pi))$

$x(t)=\dfrac{j}{2\pi}e^{j100\pi t}-\dfrac{j}{2\pi}e^{-j100\pi t}$

$=-\dfrac{1}{\pi}\left\{\dfrac{1}{2j}e^{j100\pi t}-\dfrac{1}{2j}e^{-j100\pi t}\right\}$ ← use Inverse Euler

$x(t)=-\dfrac{1}{\pi}\sin(100\pi t)$

PROBLEM 11.4:

(a) $x(t) = u(t) - u(t-4)$ is a shifted pulse

$$= \underset{\text{time-shift}}{\delta(t-2)} * \underbrace{[u(t+2) - u(t-2)]}_{\text{F.T.} = \frac{\sin(2\omega)}{\omega/2}}$$

$$X(j\omega) = e^{-j2\omega} \frac{\sin(2\omega)}{\omega/2}$$

(b) Each impulse in ω inverts to a complex exponential

$$S(j\omega) = 4\pi\delta(\omega) + 2\pi\delta(\omega - 10\pi) + 2\pi\delta(\omega + 10\pi)$$

$$s(t) = 2e^{j0} + e^{j10\pi t} + e^{-j10\pi t}$$

$$= 2 + 2\cos(10\pi t)$$

(c) $R(j\omega) = \frac{1}{2} - \frac{2}{4 + j2\omega} = \frac{1}{2} - \frac{1}{2 + j\omega}$

$$r(t) = \frac{1}{2}\delta(t) - e^{-2t}u(t)$$

(d) $y(t) = \delta(t+1) + 2\delta(t) + \delta(t-1)$

$$Y(j\omega) = e^{j\omega} + 2 + e^{-j\omega}$$

$$= 2 + 2\cos(\omega)$$

PROBLEM 11.5:

Use the derivative property:

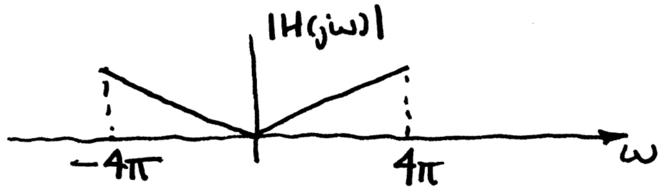
$$\frac{d}{dt}x(t) \xrightarrow{\text{F.T.}} j\omega X(j\omega)$$

Here, $x(t) = \dfrac{\sin(4\pi t)}{\pi t}$

$$\Rightarrow X(j\omega) = u(\omega+4\pi) - u(\omega-4\pi)$$

Then, $H(j\omega) = j\omega\left[u(\omega+4\pi) - u(\omega-4\pi)\right]$

$$|H(j\omega)| = |\omega| \quad \text{for } -4\pi < \omega < 4\pi$$



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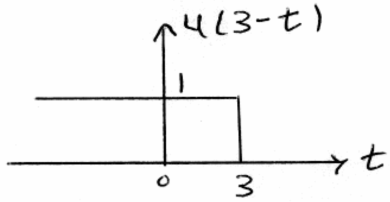
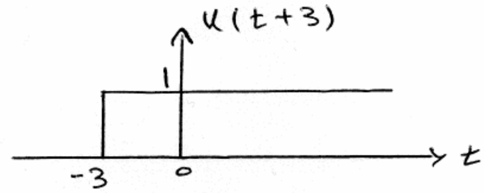
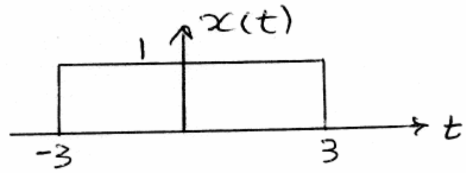
PROBLEM 11.6:

(a)





$x(t) = u(t+3)\cdot u(3-t) \longrightarrow$



$$T_0/2 = 3 \rightarrow X(j\omega) = \frac{\sin(3\omega)}{\omega/2}$$

(b) Note

t-domain $\qquad\qquad$ ω-domain

$$\sin 4\pi t \longleftrightarrow \frac{\pi}{j}\delta(\omega-4\pi) - \frac{\pi}{j}\delta(\omega+4\pi)$$

Convolution property of the Fourier Transform:

$$X(j\omega) = \frac{1}{2\pi}\left\{\frac{\pi}{j}\delta(\omega-4\pi) - \frac{\pi}{j}\delta(\omega+4\pi)\right\} * \left\{\frac{\pi}{j}\delta(\omega-50\pi) - \frac{\pi}{j}\delta(\omega+50\pi)\right\}$$

$$X(j\omega) = \frac{\pi}{2}\delta(\omega-46\pi) + \frac{\pi}{2}\delta(\omega+46\pi) - \frac{\pi}{2}\delta(\omega-54\pi) - \frac{\pi}{2}\delta(\omega+54\pi)$$

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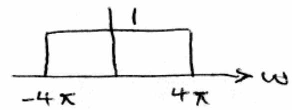
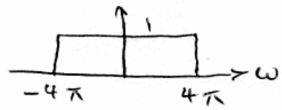
PROBLEM 11.6 (more):

(c) Note $\dfrac{\sin 4\pi t}{\pi t} \longleftrightarrow$



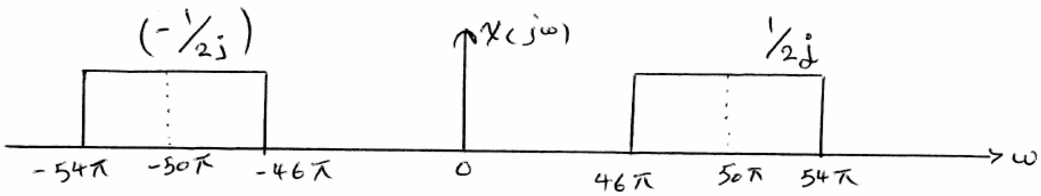
$X(j\omega) = \dfrac{1}{2\pi}\Big\{$



$\Big\} * \left\{\dfrac{\pi}{j}\delta(\omega - 50\pi) - \dfrac{\pi}{j}\delta(\omega + 50\pi)\right\}$

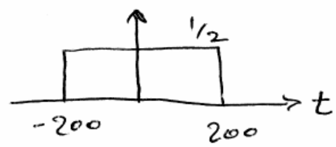
convolution

Thus

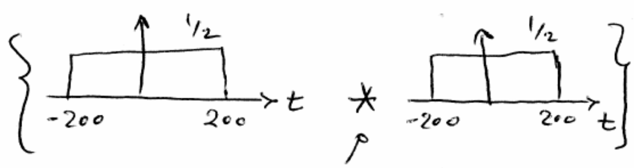


$$X(j\omega) = \begin{cases} \dfrac{1}{2j} & 46\pi \le \omega \le 54\pi \\ -1/2j & -54\pi \le \omega \le -46\pi \\ 0 & \text{else} \end{cases}$$

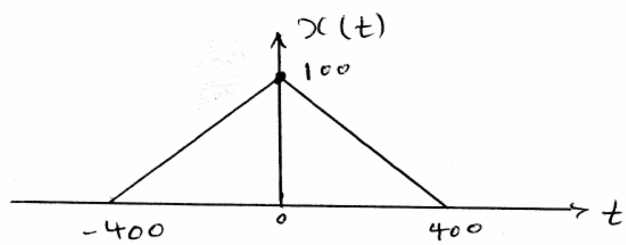
(d) Note: $\dfrac{1}{2}\dfrac{\sin(200\omega)}{\omega/2} \longleftrightarrow$



$\dfrac{\sin^2(200\omega)}{\omega^2} \longleftrightarrow$



Thus



(e) $\cos\omega \longleftrightarrow \dfrac{1}{2}\left\{\delta(t-1) + \delta(t+1)\right\}$

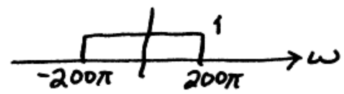
$\cos^2\omega \longleftrightarrow \dfrac{1}{2}\left\{\delta(t-1) + \delta(t+1)\right\} * \dfrac{1}{2}\left\{\delta(t-1) + \delta(t+1)\right\}$

$x(t) = \dfrac{1}{4}\left\{\delta(t-2) + \delta(t+2) + 2\delta(t)\right\}$

PROBLEM 11.7:

(a) Use derivative property: $\frac{d}{dt}x(t) \longrightarrow j\omega X(j\omega)$

F.T. of $\dfrac{\sin(200\pi t)}{\pi t}$ is a rectangle



Thus

$$X(j\omega) = \begin{cases} j10\omega & \text{if } |\omega| \le 200\pi \\ 0 & \text{if } |\omega| > 200\pi \end{cases}$$

or,

$$X(j\omega) = j10\omega\left[u(\omega + 200\pi) - u(\omega - 200\pi)\right]$$

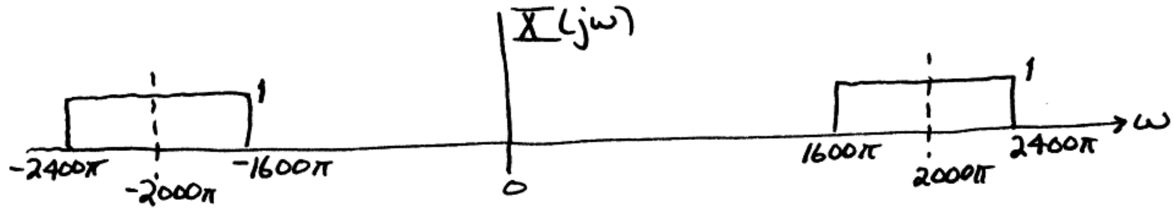
(b) Multiply by cosine $\Longrightarrow$ frequency shifting

$$x(t) = 2\,\frac{\sin(400\pi t)}{\pi t}\left\{\frac{1}{2}e^{j2000\pi t} + \frac{1}{2}e^{-j2000\pi t}\right\}.$$

F.T. is a rectangle

shift to $\omega = 2000\pi$

shift to $\omega = -2000\pi$



(c) The Fourier Transform of an impulse train in time is a (different) impulse train in frequency.

$T = 10$ secs from the definition of $x(t)$.

$$\Rightarrow \quad X(j\omega) = \frac{2\pi}{10}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi k}{10}\right)$$

spacing is $\frac{\pi}{5}$ rads.

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PROBLEM 11.8:

(a) $X(j\omega) = e^{-j3\omega}\left(\dfrac{1}{2+j\omega}\right)$ $\xleftarrow{}$ FT^{-1} is $e^{-2t}u(t)$

$$x(t) = e^{-2(t-3)}u(t-3)$$

(b) $X(j\omega) = j\omega\left(\dfrac{1}{2+j\omega}\right)$ use derivative property

$$x(t) = \frac{d}{dt}\left\{e^{-2t}u(t)\right\} = \underbrace{e^{-2t}\delta(t)}_{\text{eval @ } t=0} - 2e^{-2t}u(t)$$

$$x(t) = \delta(t) - 2e^{-2t}u(t)$$

(c) $X(j\omega) = e^{-j3\omega}\left(\dfrac{j\omega}{2+j\omega}\right)$ use time-shift on the result of (b)

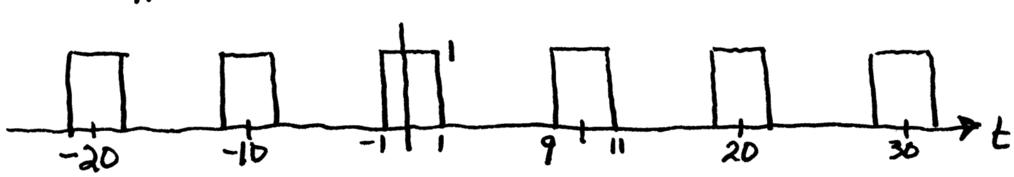
$$x(t) = \delta(t-3) - 2e^{-2(t-3)}u(t-3)$$

(d) $\dfrac{2\sin(\omega)}{\omega} = \dfrac{\sin(\omega)}{\omega/2} \xrightarrow{FT^{-1}} u(t+1) - u(t-1)$

$$\frac{2\pi}{10}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi}{10}k\right) \xrightarrow{FT^{-1}} \sum_{n=-\infty}^{\infty}\delta(t-10n)$$

Convolve: $\left[u(t+1)-u(t-1)\right] * \displaystyle\sum_{n=-\infty}^{\infty}\delta(t-10n)$

$$= \sum_{n=-\infty}^{\infty}\left[u(t+1-10n) - u(t-1-10n)\right]$$



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PROBLEM 11.9:

$h(t)$ is real $\Rightarrow$ $h^*(t) = h(t)$

If $h(t) \longrightarrow H(j\omega)$, then $h^*(t) \longrightarrow H^*(-j\omega)$

$\Rightarrow$ $H^*(-j\omega) = H(j\omega)$

Express $H(j\omega)$ in terms of its real and imaginary parts:

$$H(j\omega) = A(\omega) + j\,B(\omega)$$

Then $H^*(-j\omega) = A(-\omega) - j\,B(-\omega)$

$\Rightarrow$ $A(\omega) = A(-\omega)$ and $-B(\omega) = B(-\omega)$

The magnitude is even:

$$|H(-j\omega)| = \sqrt{A^2(-\omega) + B^2(-\omega)}$$

$$= \sqrt{A^2(\omega) + B^2(\omega)} = |H(j\omega)|$$

The phase is odd:

$$\angle H(-j\omega) = \mathrm{Tan}^{-1}\left\{\frac{B(-\omega)}{A(-\omega)}\right\}$$

$$= \mathrm{Tan}^{-1}\left\{\frac{-B(\omega)}{A(\omega)}\right\}$$

$$= -\mathrm{Tan}^{-1}\left\{\frac{B(\omega)}{A(\omega)}\right\} = -\angle H(j\omega)$$

Recall that the tangent function is an <u>ODD</u> function.

PROBLEM 11.10:

$$X(j\omega) = 5\left(\frac{1}{3+j\omega}\right)\left(\frac{1}{3+j\omega}\right)$$

$$\Rightarrow x(t) = 5\, e^{-3t}u(t) * e^{-3t}u(t)$$

$$= 5t\, e^{-3t}u(t)$$

Consult Problem 9.8 for this convolution

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PROBLEM 11.11:

(a)

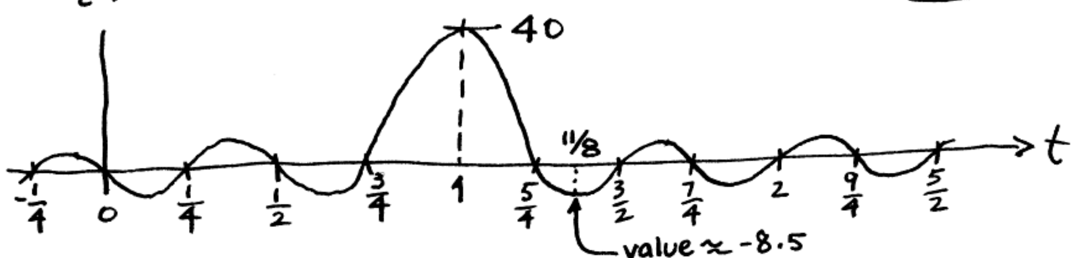
$h(t)$ is a shifted "sinc": $10\frac{\sin 4\pi t}{\pi t}$ shifted by 1 sec.

For the "sinc", the zero crossings are at $\frac{1}{4}, \frac{1}{2}, \frac{3}{4} \ldots$

At $t=0$, the "sinc" is $10 \cdot 4\pi/\pi = 40$.

$$\lim_{t\to 0} 10\frac{\sin 4\pi t}{\pi t} = \lim_{t\to 0} 10\frac{4\pi t}{\pi t} = 40$$

Use small angle approximation $\sin(\epsilon) \approx \epsilon$

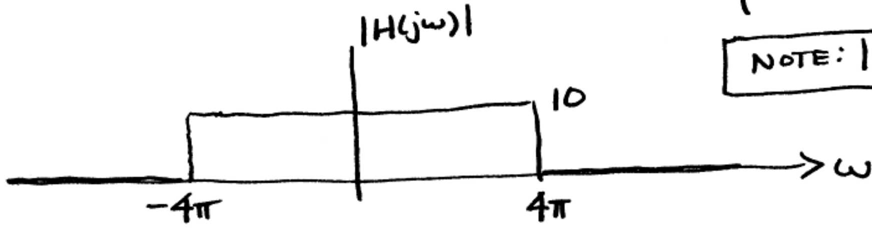


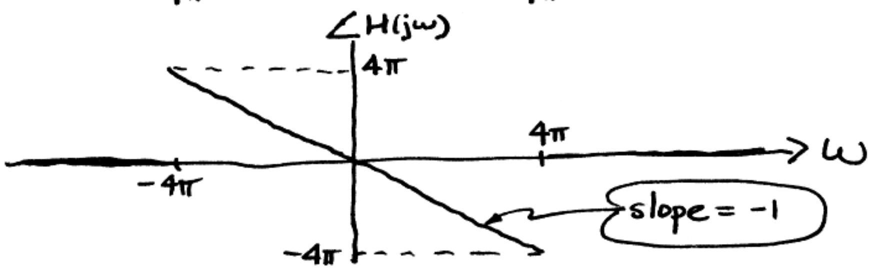
The smaller peaks are approximately halfway between the zero crossings. For example, the value at $t = \frac{11}{8}$ secs is $10\frac{\sin(4\pi \cdot 3/8)}{\pi(3/8)} = 10\frac{\sin(3\pi/2)}{3\pi/8} = -\frac{80}{3\pi} \approx -8.5$

(b) The F.T. of a "sinc" is a rectangular pulse.
Time-shifting by 1 sec. corresponds to $e^{-j\omega}$

$$\Rightarrow H(j\omega) = 10e^{-j\omega}\{u(\omega+4\pi)-u(\omega-4\pi)\} = \begin{cases} 10e^{-j\omega} & |\omega| \le 4\pi \\ 0 & |\omega| > 4\pi \end{cases}$$

NOTE: $|e^{-j\omega}| = 1$





NOTE-1: The angle of 0 is taken to be zero.

NOTE-2: If the phase were plotted in MATLAB or evaluated with an ArcTangent function, it would exhibit jumps of 2π because ArcTan always gives an answer between $-\pi$ and $+\pi$.

PROBLEM 11.12:

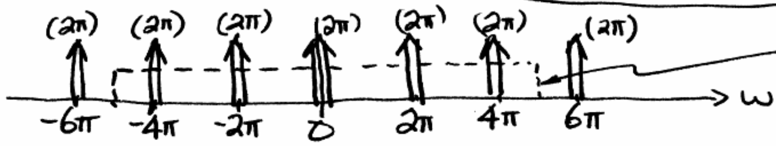
(a) The F.T. of an impulse train is another impulse train.

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-n) = \sum_{k=-\infty}^{\infty} e^{j2\pi kt}$$

Each complex exp transforms to $2\pi\delta(\omega - k\omega_0)$

$$X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k)$$

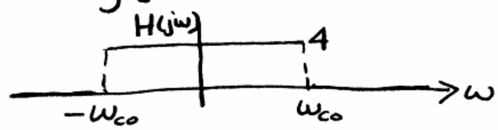
NOTE: $\omega_0 = 2\pi$ rad/s



DASHED LINE is $H(j\omega)$ from part (b)

(b) The F.T. of a "sinc" is a rectangle

$$h(t) = 4\,\frac{\sin(\omega_{co} t)}{\pi t} \longrightarrow$$



(c) $Y(j\omega)$ will consist of 5 impulses because $H(j\omega)=0$ when $|\omega| > 5\pi$

$$Y(j\omega) = 8\pi\delta(\omega) + 8\pi\delta(\omega-2\pi) + 8\pi\delta(\omega+2\pi) + 8\pi\delta(\omega-4\pi) + 8\pi\delta(\omega+4\pi)$$

Each $\delta(\omega - ?)$ will inverse F.T. to an exponential.

$$y(t) = 4 + 4e^{j2\pi t} + 4e^{-j2\pi t} + 4e^{j4\pi t} + 4e^{-j4\pi t}$$

$$y(t) = 4 + 8\cos(2\pi t) + 8\cos(4\pi t) \qquad \text{for } \underline{\text{all}}\ t$$

(d) If you want only the constant term then put the cutoff frequency of the filter below 2π rad/s, but keep it greater than ZERO.

$$\boxed{0 < \omega_{co} < 2\pi}$$

In this case, $Y(j\omega) = 8\pi\delta(\omega)$

$$\Rightarrow y(t) = 4 \qquad \boxed{c = 4}$$

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PROBLEM 11.13:

(a) Since the system is LINEAR, the two inputs can be treated separately and then combined.

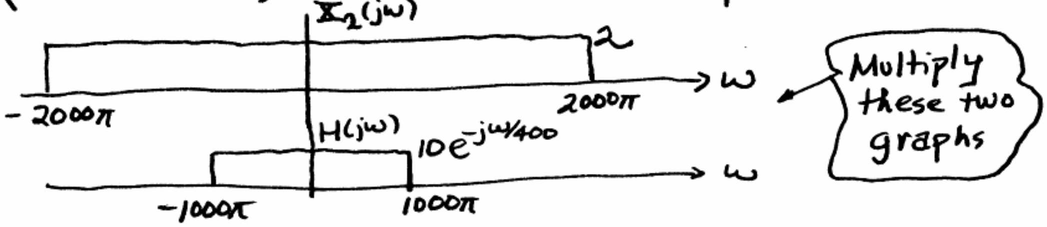
- For the cosine input, the output will be a cosine with a new magnitude and phase. We evaluate $H(j\omega)$ at the input frequency: $\omega = 200\pi$ rad/s.

$$H(j200\pi) = 10e^{-j(200\pi)(0.0025)} = 10e^{-j0.5\pi}$$

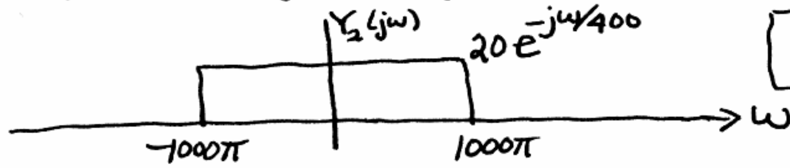
Call this output $y_1(t)$: $\quad y_1(t) = 10\cos(200\pi t - \pi/2)$

- For the "sinc" input, take the F.T. of the input, then multiply by $H(j\omega)$ and then inverse transform.

$$\text{F.T.}\left\{2\,\frac{\sin(2000\pi t)}{\pi t}\right\} = \text{Rectangular Shape: width of } 4000\pi \text{ rad/s. amp} = 2$$



Thus $Y_2(j\omega) = H(j\omega)X_2(j\omega)$ is also a rectangle



$\boxed{\frac{1}{400} = 0.0025\text{s}}$

The inverse F.T. of this rectangle is a SHIFTED "sinc"

$$y_2(t) = 20\,\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

Finally, the total output is the sum of $y_1(t)$ & $y_2(t)$

$$y(t) = 10\cos(200\pi t - \pi/2) + 20\,\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

We have used SUPERPOSITION to do this part.

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(b) Use SUPERPOSITION again. Two of the inputs are the same, so we don't have to rework them. We only need to consider the input $x_3(t) = \cos(3000\pi t)$.
For a cosine input, we must evaluate $H(j\omega)$ at the input frequency; in this case, at $\omega = 3000\pi$.

$H(j3000\pi) = 0 \quad \Rightarrow$ NO OUTPUT, i.e. $y_3(t) = 0$.

So, the answer is the same as part (a) !

(c) Again, use SUPERPOSITION. We already know the output for $x_1(t) = \cos(200\pi t)$

$$y_1(t) = 10\cos(200\pi t - \pi/2)$$

We need to find the output for $x_4(t) = 2\delta(t)$.
Thus we need the impulse response
But this is just the inverse F.T. of $H(j\omega)$
And we already know that is a shifted "sinc"

$$y_4(t) = 2h(t) = 20\,\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

Finally,

$$y(t) = y_1(t) + y_4(t)$$
$$= 10\cos(200\pi t - \pi/2) + 20\,\frac{\sin(1000\pi(t - 1/400))}{\pi(t - 1/400)}$$

It is very interesting to see that all three parts have the same answer. Why? Because the filter is an ideal LPF, so only the part of the input signal between -1000π and $+1000\pi$ matters. For example, in part (c) the F.T. of $2\delta(t)$ is $X_4(j\omega) = 2$ for all ω, but only the part for $|\omega| < 1000\pi$ rad/s matters. Over that range the "sinc" input of part (a) is the same

(d) Superposition simplifies the work.

PROBLEM 11.14:

The periodic input to the above system is defined by the equation:

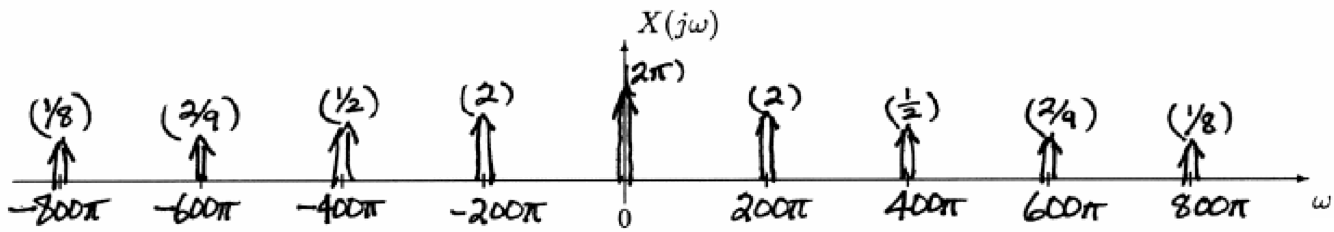
$$x(t) = \sum_{k=-4}^{4} a_k e^{j200\pi kt}, \quad \text{where} \quad a_k = \begin{cases} \dfrac{1}{\pi|k|^2} & k \neq 0 \\ 1 & k = 0 \end{cases}$$

$a_1 = 1/\pi$
$a_2 = 1/4\pi$
$a_3 = 1/9\pi$
$a_4 = 1/16\pi$

(a) Determine the Fourier transform of the periodic signal $x(t)$. Give a formula and then plot it on the graph below.

$$X(j\omega) = \sum_{k=-4}^{4} 2\pi a_k \delta(\omega - 200\pi k)$$

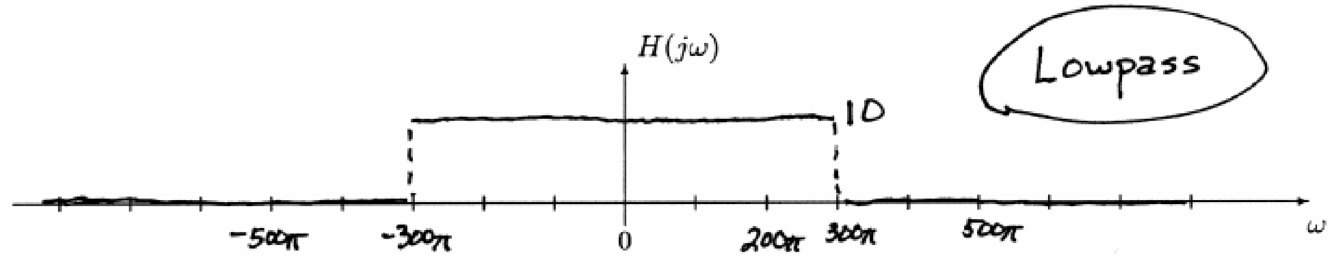
$\omega_0 = 200\pi$



(b) The frequency response of the LTI system is given by the following equation:

$$H(j\omega) = \begin{cases} 10 & |\omega| \leq 300\pi \\ 0 & |\omega| > 300\pi \end{cases}.$$

Plot this function on the graph below using the same frequency scale as the plot in part (a). *Note carefully what type of filter (i.e., lowpass, bandpass, highpass) this is.*



Lowpass

(c) Write an equation for $y(t)$.

$$Y(j\omega) = H(j\omega)X(j\omega) = 20\pi\delta(\omega) + 20\delta(\omega - 200\pi) + 20\delta(\omega + 200\pi)$$

Invert: $y(t) = 10 + \underbrace{\frac{10}{\pi} e^{j200\pi t} + \frac{10}{\pi} e^{-j200\pi t}}_{\text{use Euler's inverse formula}}$

$$y(t) = 10 + \frac{20}{\pi}\cos(200\pi t)$$

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PROBLEM 11.15:

(a)

$$x(t) = \begin{cases} 1-|t| & \text{for } |t|<1 \\ 0 & \text{elsewhere} \end{cases}$$

Let $g(t) = \frac{d}{dt}x(t)$

Then $G(j\omega) = j\omega X(j\omega) \Rightarrow X(j\omega) = \frac{G(j\omega)}{j\omega}$

$$g(t) = [u(t+1) - u(t)] - [u(t) - u(t-1)]$$

$$\Rightarrow G(j\omega) = e^{j\omega/2}\frac{\sin(\omega/2)}{\omega/2} - e^{-j\omega/2}\frac{\sin(\omega/2)}{\omega/2}$$

$$= \left(2j\sin(\omega/2)\right)\left(\frac{\sin(\omega/2)}{\omega/2}\right)$$

$$X(j\omega) = 2j\frac{\sin^2(\omega/2)}{(\omega/2)(j\omega)} = \frac{\sin^2(\omega/2)}{(\omega/2)^2}$$

(b) Let $f(t) = \frac{d^2}{dt^2}x(t) = \delta(t+1) - 2\delta(t) + \delta(t-1)$

$$F(j\omega) = (j\omega)^2 X(j\omega)$$

$$\Rightarrow X(j\omega) = \frac{F(j\omega)}{(j\omega)^2}$$

$$F(j\omega) = e^{j\omega} - 2 + e^{-j\omega} = 2\cos(\omega) - 2$$

$$\Rightarrow X(j\omega) = \frac{2\cos(\omega) - 2}{-\omega^2} = \frac{2(\cos(\omega) - 1)}{\omega^2}$$

The results in part (a) and (b) are the same, but a trigonometric is needed.

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PROBLEM 11.16:

$x(t)$ is real $\Rightarrow$ $x^*(t) = x(t)$

If $x(t) \longrightarrow X(j\omega)$, then $x^*(t) \longrightarrow X^*(-j\omega)$

$\Rightarrow X^*(-j\omega) = X(j\omega)$

Express $X(j\omega)$ in terms of its real and imaginary parts:

$$X(j\omega) = A(\omega) + jB(\omega)$$

Then $X^*(-j\omega) = A(-\omega) - jB(-\omega)$

$\Rightarrow A(\omega) = A(-\omega)$ and $-B(\omega) = B(-\omega)$

(a) The magnitude is even:

$$|X(-j\omega)| = \sqrt{A^2(-\omega) + B^2(-\omega)}$$
$$= \sqrt{A^2(\omega) + B^2(\omega)} = |X(j\omega)|$$

(b) The phase is odd:

$$\angle X(-j\omega) = \text{Tan}^{-1}\left\{\frac{B(-\omega)}{A(-\omega)}\right\}$$
$$= \text{Tan}^{-1}\left\{\frac{-B(\omega)}{A(\omega)}\right\}$$
$$= -\text{Tan}^{-1}\left\{\frac{B(\omega)}{A(\omega)}\right\} = -\angle X(j\omega)$$

Recall that the tangent function is an <u>ODD</u> function.

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PROBLEM 11.17:

Define $s(t) = u(t) - \frac{1}{2}$

$$\Rightarrow \quad S(j\omega) = U(j\omega) - \pi\delta(\omega)$$

Since $s(t)$ is an odd function: $s(-t) = -s(t)$

and $s(-t) \xrightarrow{FT} S(-j\omega)$

$$S(-j\omega) = -S(j\omega)$$

$$\Rightarrow \quad U(-j\omega) \underbrace{-\pi\delta(-\omega)}_{=\pi\delta(\omega)} = -U(j\omega) + \pi\delta(\omega)$$

$$U(-j\omega) = -U(j\omega) + 2\pi\delta(\omega)$$

Thus, if we assume $U(j\omega) = \frac{1}{j\omega} + K\delta(\omega)$

$$\frac{1}{-j\omega} + K\delta(-\omega) = -\frac{1}{j\omega} - K\delta(\omega) + 2\pi\delta(\omega)$$

$$\Rightarrow \quad 2K\delta(\omega) = 2\pi\delta(\omega)$$

$$\Rightarrow \quad K = \pi$$

Note: $\delta(-\omega) = \delta(\omega)$, i.e. $\delta(\,)$ is <u>even</u>