$$|p_n - p| \le k^n \max\{p_0 - a, b - p_0\}$$

or

$$|p_n - p| \le \frac{k^n}{1-k} |p_1 - p_0| \quad \text{for all } n \ge 1.$$

Thus, the rate of convergence of the fixed point iteration is $O(k^n)$.

Ex. Considering the iteration functions in previous example (#6/64).

(a) $g(x) = x^3 - 1$ $\quad g(1) = 0 \quad g(2) = 7$

Thus, $1 \le g(x) \le 2$ is NOT satisfied

Besides, $|g'(x)| > 1$

Although this does not guarrantee diverge, that is what happens in this case

(b) $g(x) = \sqrt[3]{x+1}$

$$\sqrt[3]{2} \le g(x) \le \sqrt[3]{3} \Rightarrow 1 \le g(x) \le 2$$

$$g'(x) = \frac{1}{3}(x+1)^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{(x+1)^2}} \text{ - decreasing}$$

$$\Rightarrow |g'(x)| \le \frac{1}{3} \frac{1}{\sqrt[3]{2^2}} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{4}} = 0.209986841\overline{6}$$

$\Rightarrow$ The fixed-point iteration should converge with rate $(0.21)^n$.