$$\frac{e^h + e^{-h} - 2\cosh}{h} = \frac{\cancel{2} + 2\frac{h^2}{2!} + 2\frac{h^4}{4!} - \cancel{2} + 2\cdot\frac{h^2}{2!} - 2\cdot\frac{h^4}{4!} + \cdots}{h}$$

$$\approx \frac{4\cdot\frac{h^2}{2!}}{h} = 2h$$

$\Rightarrow$ The rate of convergence is $O(h)$

4) Solutions of equations in one variable

Ex: Find the number of iterations needed to achieve an approximation with accuracy 10^{-5} to the solution of

$$x\cos x - 2x^2 + 3x - 1 = 0$$

in the interval $[0.2, 0.3]$ using the bisection method. Use the formula:

$$|p_n - p| \leq \frac{b-a}{2^n}$$

Solution:

$$\frac{0.3-0.2}{2^n} < 10^{-5} \rightarrow \frac{0.1}{2^n} < \frac{1}{10^5}$$

$$\frac{1}{2^n}\cdot\frac{1}{10} < \frac{1}{10^5}$$

$$\frac{1}{2^n} < \frac{1}{10^4}$$

$$2^n > 10^4$$