

2.3 Newton's Method

I Newton's method

- Newton's method is one of the most powerful numerical methods for solving root-finding problems.

1) Newton's Method from Taylor's expansion

Suppose $f \in C^2[a, b]$. Let $f(p) = 0$.

Let $\bar{x} \approx p$, so that

a) $|p - \bar{x}|$ - small

b) $f'(\bar{x}) \neq 0$

Consider the Taylor's polynomial of $f(x)$ expanded around \bar{x}

$$f(x) = f(\bar{x}) + f'(\bar{x})(x - \bar{x}) + f''(\xi(x)) \frac{(x - \bar{x})^2}{2!}$$

where $\xi(x)$ is a point between x and \bar{x} .

Set $x = p$, note $f(p) = 0$

$$0 = f(\bar{x}) + f'(\bar{x})(p - \bar{x}) + \underbrace{f''(\xi(p)) \frac{(p - \bar{x})^2}{2!}}_{\text{ignore}}$$

$$\bar{x} f'(\bar{x}) - f(\bar{x}) \approx p f'(\bar{x})$$