

Ex 1/153 A natural cubic spline is given as

$$S(x) = \begin{cases} S_0(x) = 1 + B(x-1) - D(x-1)^3 & 1 \leq x \leq 2 \\ S_1(x) = 1 + b(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3 & 2 \leq x \leq 3 \end{cases}$$

Find B, D, b, d .

$$S'(x) = \begin{cases} S_0'(x) = B - 3D(x-1)^2 & 1 \leq x \leq 2 \\ S_1'(x) = b - \frac{6}{4}(x-2) + 3d(x-2)^2 & 2 \leq x \leq 3 \end{cases}$$

$$S''(x) = \begin{cases} S_0''(x) = -6D(x-1) & 1 \leq x \leq 2 \\ S_1''(x) = -\frac{6}{4} + 6d(x-2) & 2 \leq x \leq 3 \end{cases}$$

$$S_0(2) = S_1(2): 1 + B - D = 1 \Rightarrow B = D$$

$$S_0'(2) = S_1'(2): B - 3D = b \Rightarrow -2B = b$$

$$S_0''(2) = S_1''(2): -6D = -\frac{6}{4} \Rightarrow D = \frac{1}{4}$$

$$\Rightarrow B = \frac{1}{4}$$

$$\Rightarrow b = -\frac{1}{2}$$

$$S_0''(1) = 0 \text{ - automatic}$$

$$S_1''(3) = -\frac{6}{4} + 6d = 0 \Rightarrow 6d = \frac{6}{4}$$

$$\Rightarrow d = \frac{1}{4}$$