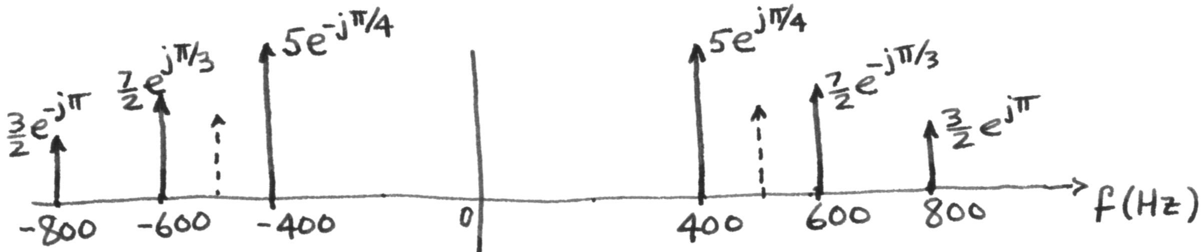
PROBLEM 3.1:

(a) There are 3 components

$$10\cos(800\pi t + \pi/4) = \Re e\{10e^{j\pi/4}e^{j800\pi t}\} \qquad \text{freq} = 400\text{ Hz}$$

$$7\cos(1200\pi t - \pi/3) = \Re e\{7e^{-j\pi/3}e^{j1200\pi t}\} \qquad \text{freq} = 600\text{ Hz}$$

$$-3\cos(1600\pi t) = \Re e\{3e^{j\pi}e^{j1600\pi t}\} \qquad \text{freq} = 800\text{ Hz}$$



SPECTRUM

(b) $x(t)$ is periodic because there is a fundamental frequency $f = 200$ Hz that divides all 3 freqs. The period is the fundamental period $= 1/200$ sec

(c)

$$5\cos(1000\pi t + \pi/2) = \Re e\{5e^{j\pi/2}e^{j1000\pi t}\} \qquad \text{freq} = 500\text{ Hz}$$

The spectrum will have two additional lines at $f = -500$ Hz and $f = 500$ Hz

$\frac{5}{2}e^{-j\pi/2}$

component is $\frac{5}{2}e^{j\pi/2}$

The dotted lines in the sketch above show where these two lines will be.

Yes, $y(t)$ is periodic. The fundamental frequency is now $f_0 = 100$ Hz because it has to divide into 500 Hz as well as 400, 600 and 800.

The period is now $\frac{1}{100}$ sec.

PROBLEM 3.2:

(a) Read values from the graph:

$$x(t) = 4e^{j\pi/2} e^{-j2\pi(175)t} + 7e^{j\pi/3} e^{-j2\pi(50)t} + 11e^{j0t}$$
$$+ 4e^{-j\pi/2} e^{j2\pi(175)t} + 7e^{-j\pi/3} e^{+j2\pi 50 t}$$

- Combine the positive & negative freqs:

$$x(t) = 8\cos(2\pi(175)t - \pi/2) + 14\cos(2\pi(50)t - \pi/3) + 11$$

(b) Cosine at $\omega = 2\pi(175)$ has period $= \frac{2\pi}{\omega} = \frac{1}{175}$

Cosine @ $\omega = 2\pi(50) \Rightarrow$ period $= \frac{1}{50}$

11 is a constant $\Rightarrow$ freq $= 0 \Rightarrow$ any period.

Need to find common period:

$\Rightarrow$ Solve $\ell_1\left(\frac{1}{50}\right) = \ell_2\left(\frac{1}{175}\right)$ $\quad \ell_1$ & ℓ_2 integers

$\Rightarrow \frac{\ell_1}{\ell_2} = \frac{50}{175} = \frac{2}{7}$ ← REDUCED TO LOWEST TERMS

$\therefore$ min period $= \frac{2}{50} = \frac{1}{25}$ sec.

(c) $y(t) = A\cos(\omega_0 t + \varphi)$

$= \Re e\{A e^{j\varphi} e^{j\omega_0 t}\}$

$= \frac{1}{2}A e^{j\varphi} e^{j\omega_0 t} + \frac{1}{2}A e^{-j\varphi} e^{-j\omega_0 t}$

$Z_1 = \frac{1}{2}A e^{j\varphi}$ $\qquad Z_1^* = \frac{1}{2}A e^{-j\varphi}$ $\qquad$ NEGATIVE FREQ

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PROBLEM 3.3:

(a)

$$x(t) = \sin^3(27\pi t)$$

$$= \left(\frac{1}{2j}e^{j27\pi t} - \frac{1}{2j}e^{-j27\pi t}\right)^3$$

$$= \frac{j}{8}\left(e^{j81\pi t} - 3e^{j54\pi t - j27\pi t} + 3e^{j27\pi t - j54\pi t} - e^{-j81\pi t}\right).$$

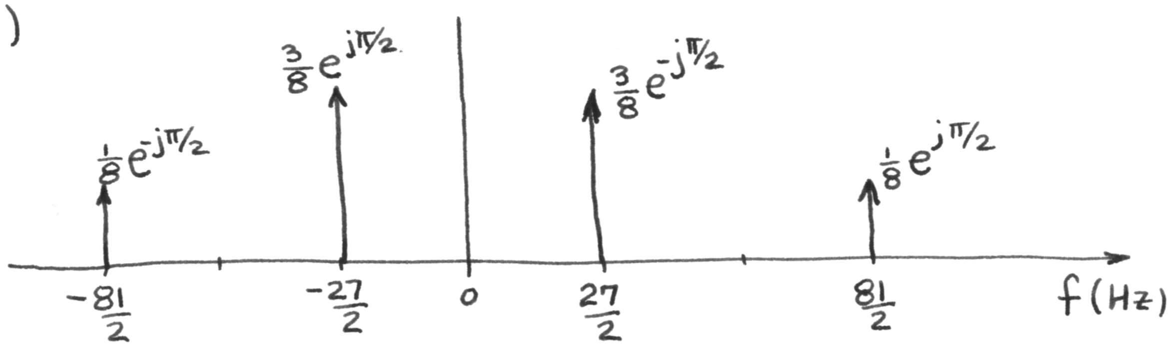
$$x(t) = \frac{1}{8}e^{j\pi/2}e^{j81\pi t} + \frac{3}{8}e^{-j\pi/2}e^{j27\pi t} + \frac{3}{8}e^{j\pi/2}e^{-j27\pi t} + \frac{1}{8}e^{-j\pi/2}e^{-j81\pi t}$$

(b) $e^{\pm j81\pi t}$ has period $= \frac{2\pi}{81\pi} = \frac{2}{81}$

$e^{\pm j27\pi t}$ has period $= 2\pi/27\pi = 2/27. = 3(2/81).$

$\therefore$ period $= \frac{2}{27}$ sec.

(c)



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PROBLEM 3.4:

(a) $$x(t) = \Re e\{Ae^{j2\pi(f_c - f_\Delta)t}\} + \Re e\{Be^{j2\pi(f_c+f_\Delta)t}\}$$

$$= \Re e\{\underbrace{\left(Ae^{-j2\pi f_\Delta t} + Be^{j2\pi f_\Delta t}\right)e^{j2\pi f_c t}}_{\tilde{x}(t)}\}$$

(b) $$\tilde{x}(t) = \left((B+A)\cos(2\pi f_\Delta t) + j(B-A)\sin(2\pi f_\Delta t)\right)e^{j2\pi f_c t}$$

$$\Re e\{\tilde{x}(t)\} = (B+A)\cos(2\pi f_\Delta t)\cos(2\pi f_c t) - (B-A)\sin(2\pi f_\Delta t)\sin(2\pi f_c t)$$

$$\Rightarrow \quad C = B+A$$
$$D = -(B-A) = A-B$$

when $A=B=1$, we get $C=2$ and $D=0$ so the sine terms drop out.

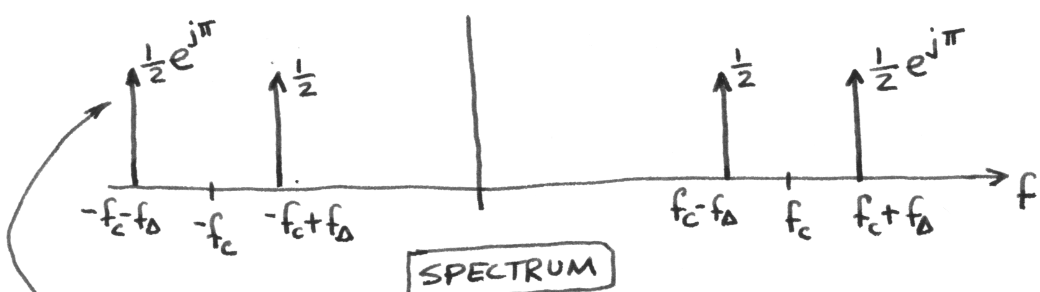
(c) Want $D=2$ and $C=0$

$$\left.\begin{array}{l} B+A=0 \\ A-B=2 \end{array}\right\} \Rightarrow \quad A=1 \text{ and } B=-1$$

Return to the original expression for $x(t)$

$$x(t) = \tfrac{1}{2}e^{j2\pi(f_c-f_\Delta)t} + \tfrac{1}{2}e^{-j2\pi(f_c-f_\Delta)t} - \tfrac{1}{2}e^{j2\pi(f_c+f_\Delta)t} - \tfrac{1}{2}e^{-j2\pi(f_c+f_\Delta)t}$$

There are 4 terms



NOTE: $-\frac{1}{2} = \frac{1}{2}e^{j\pi}$

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PROBLEM 3.5:

$x(t)$ has three components:

$10 = 10e^{j0} \Rightarrow$ freq $= 0$ & period = anything

$20\cos(2\pi(100)t + \pi/4) = \Re e\{20\, e^{j\pi/4} e^{j200\pi t}\}$

freq $= 100$ Hz $\Rightarrow$ period $= 1/100$ sec

$10\cos(2\pi(250)t) = \Re e\{10\, e^{j500\pi t}\}$ freq $= 250$ Hz, period $= 1/250$ sec.

(a) Find the fundamental frequency first:

f_0 must divide both 100 Hz and 250 Hz.

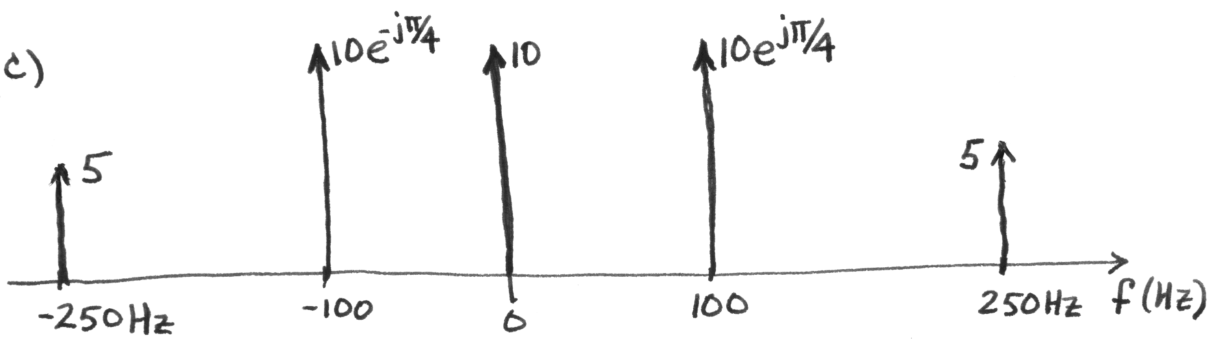
$\Rightarrow f_0 = 50$ Hz $\Rightarrow T_0 = 1/50$ sec

$X_0 = 10, \quad X_1 = 0, \quad X_2 = 20\, e^{j\pi/4}, \quad X_3 = 0, \quad X_4 = 0$

$X_5 = 10, \quad X_6 = 0, \quad X_7 = 0$, etc.

(b) We have already determined the fundamental period $T_0 = 1/50$ sec, so $x(t)$ is periodic.

(c)



NOTE: we use $\frac{1}{2}X_i$ because $\Re e\{X_i\} = \frac{1}{2}X_i + \frac{1}{2}X_i^*$

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PROBLEM 3.6:

$$x(t) = [12 + 7\sin(\pi t - \pi/3)]\cos 13\pi t$$

(a) $$x(t) = \left[12 + \frac{7}{2j}e^{j(\pi t - \pi/3)} - \frac{7}{2j}e^{-j(\pi t - \pi/3)}\right]\left(\frac{1}{2}e^{j13\pi t} + \frac{1}{2}e^{-j13\pi t}\right)$$

Multiply it all out:

$$x(t) = 6e^{j13\pi t} + 6e^{-j13\pi t} + \frac{7}{4}e^{-j\pi/2}e^{-j\pi/3}e^{j14\pi t} + \frac{7}{4}e^{-j5\pi/6}e^{-j12\pi t}$$
$$+ \frac{7}{4}e^{j\pi/2}e^{j\pi/3}e^{j12\pi t} + \frac{7}{4}e^{j5\pi/6}e^{-j14\pi t}$$

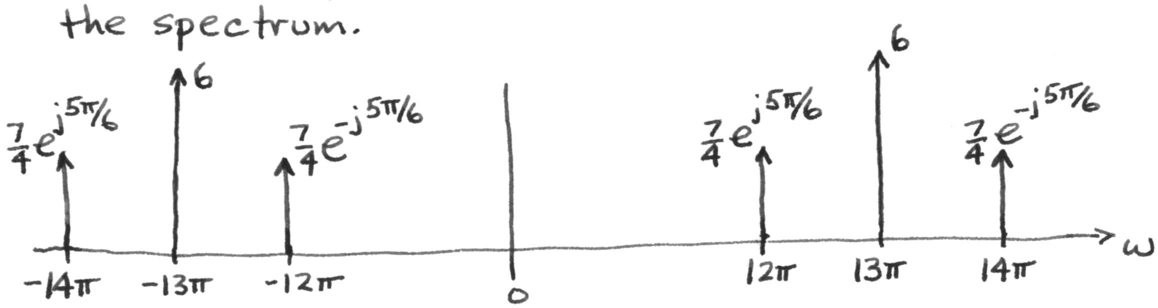
COMBINE

$$x(t) = 12\cos(13\pi t) + \frac{7}{2}\cos(14\pi t - 5\pi/6) + \frac{7}{2}\cos(12\pi t + 5\pi/6)$$

$\omega_1 = 12\pi$ rad/sec $\quad$ $\omega_2 = 13\pi$ rad/s $\quad$ $\omega_3 = 14\pi$ rad/s

$A_1 = 7/2 \quad \varphi_1 = 5\pi/6$ $\quad$ $A_2 = 12 \quad \varphi_2 = 0$ $\quad$ $A_3 = \frac{7}{2} \quad \varphi_3 = -\frac{5\pi}{6}$

(b) use the complex exponential form to sketch the spectrum.



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PROBLEM 3.7:

$$x(t) = \cos[(\omega_1+\omega_2)t] + \cos[(\omega_2-\omega_1)t]$$

(a) If $x(t)$ is to be periodic, we need to find a fundamental frequency ω_0 that divides both $(\omega_1+\omega_2)$ and $(\omega_2-\omega_1)$. That is

$$\left.\begin{aligned} \omega_1+\omega_2 &= \ell_1\omega_0 \\ \omega_2-\omega_1 &= \ell_2\omega_0 \end{aligned}\right\}$$ where ℓ_1 & ℓ_2 are integers. Also, $\ell_2 < \ell_1$

$$\Rightarrow \frac{\omega_2-\omega_1}{\omega_2+\omega_1} = \frac{\ell_2}{\ell_1} < 1$$

Thus, the ratio of the sum and difference frequencies must be a rational number.

(b) Now solve for ω_1/ω_2 and show that it must be rational.

$$(\omega_2-\omega_1)\ell_1 = (\omega_2+\omega_1)\ell_2$$

$$\omega_2(\ell_1-\ell_2) = \omega_1(\ell_1+\ell_2)$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{\ell_1-\ell_2}{\ell_1+\ell_2}$$

This is a ratio of integers

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{m_2}{m_1} < 1$$ where m_1 & m_2 are integers.

From above:

$$\begin{aligned} 2\omega_2 &= (\ell_1+\ell_2)\omega_0 \\ 2\omega_1 &= (\ell_1-\ell_2)\omega_0 \end{aligned} \Rightarrow \begin{aligned} \omega_2 &= \tfrac{1}{2}(\ell_1+\ell_2)\omega_0 \\ \omega_1 &= \tfrac{1}{2}(\ell_1-\ell_2)\omega_0 \end{aligned}$$

$\Rightarrow \omega_1$ & ω_2 are integer multiples of $\frac{1}{2}\omega_0$

PROBLEM 3.8:

$x(t)$ has four components:

$2 = 2e^{j0} \Rightarrow$ freq $= 0$, period $=$ anything

$4\cos(40\pi t - \pi/5) = \Re e\{4e^{-j\pi/5}e^{j40\pi t}\}$ freq $= 20$ Hz, period $= 1/20$ sec.

$3\sin(60\pi t) = 3\cos(60\pi t - \pi/2)$
$= \Re e\{3e^{-j\pi/2}e^{j60\pi t}\}$ freq $= 30$ Hz, period $= 1/30$ sec.

$4\cos(120\pi t - \pi/3) = \Re e\{4e^{-j\pi/3}e^{j120\pi t}\}$ freq $= 60$ Hz, period $= 1/60$ sec.

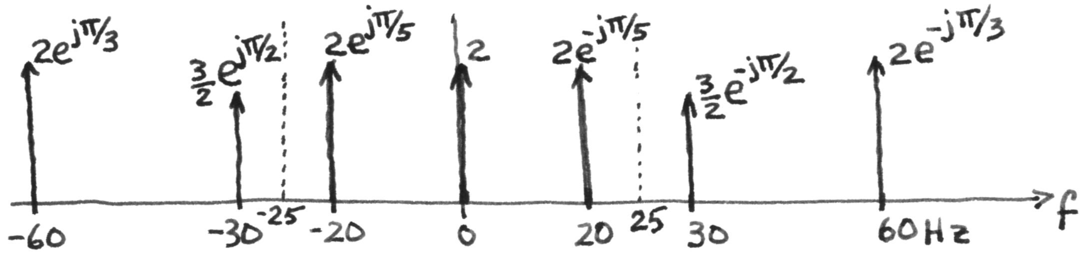
(a) The fundamental period is the smallest time that is exactly divisible by $1/20$, $1/30$ and $1/60$.

$\Rightarrow T_0 = 1/10$ sec because $T_0 = 2(1/20) = 3(1/30) = 6(1/60)$

$\Rightarrow f_0 = 10$ Hz $\Rightarrow \omega_0 = 2\pi f_0 = 20\pi$ rad/sec

$X_0 = 2$, $X_2 = 4e^{-j\pi/5}$, $X_3 = 3e^{-j\pi/2}$, $X_6 = 4e^{-j\pi/3}$

(b)



NOTE: since $\Re e\{X_i\} = \frac{1}{2}X_i + \frac{1}{2}X_i^*$ we use $\frac{1}{2}X_i$ in the plot.

(c) $y(t) = x(t) + 10\cos(50\pi t - \pi/6)$

equals $\Re e\{10e^{-j\pi/6}e^{j50\pi t}\}$ freq $= 25$ Hz, period $= 1/25$ sec

The new period must be divisible by $1/10$ & $1/25$

$\Rightarrow T_0 = 1/5$ because $T_0 = 5(1/25)$ and $2(1/10)$

$\Rightarrow f_0 = 5$ Hz $\Rightarrow \omega_0 = 2\pi f_0 = 10\pi$ rad/sec

We must re-index the X_i because ω_0 is lower.

$Y_0 = 2$, $Y_4 = X_2$, $Y_5 = 10e^{-j\pi/6}$, $Y_6 = X_3$, $Y_{12} = X_6$

There will be one new pair of lines at $f = \pm 25$ Hz

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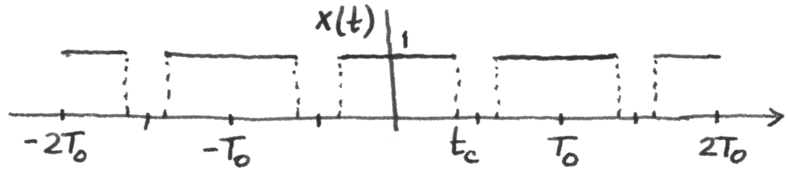
PROBLEM 3.9:

(a)



(b) $$X_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\,dt = \frac{1}{T_0}\int_{-t_c}^{t_c} 1\,dt = \frac{2t_c}{T_0}$$

(c) $$X_k = \frac{2}{T_0}\int_{-t_c}^{t_c} 1\, e^{-jk\omega_0 t}\,dt = \frac{2}{T_0}\left.\frac{e^{-jk\omega_0 t}}{(-jk\omega_0)}\right|_{-t_c}^{t_c}$$

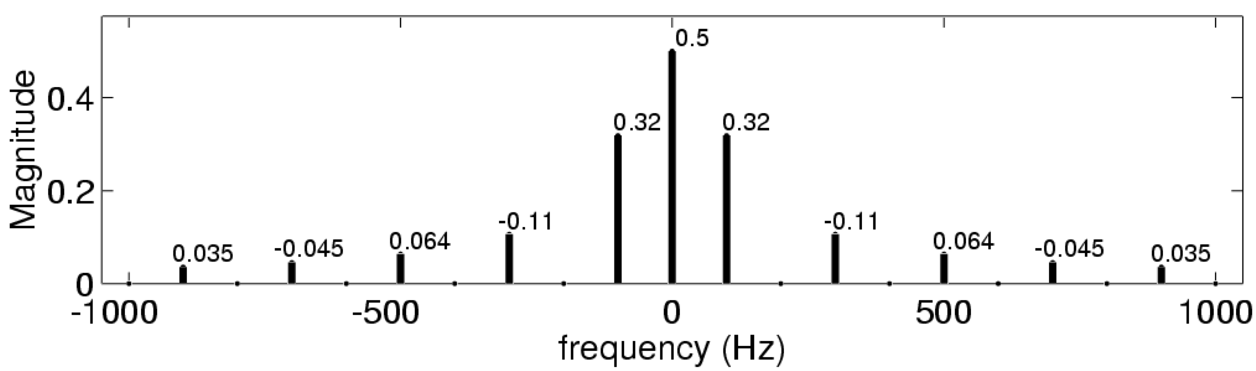
$$= \frac{2}{T_0}\,\frac{e^{-jk\omega_0 t_c} - e^{jk\omega_0 t_c}}{(-jk\omega_0)} = \frac{4}{k\omega_0 T_0}\sin(k\omega_0 t_c)$$

(d) $t_c = T_0/4$

In this case, the "sine" term is $\sin\left(\frac{2\pi k}{T_0}\cdot\frac{T_0}{4}\right) = \sin\left(\frac{\pi k}{2}\right)$

Therefore, $X_k = 0$ for k even and $k \neq 0$.

Part (d): Spectrum for $t_c = T_0/4$



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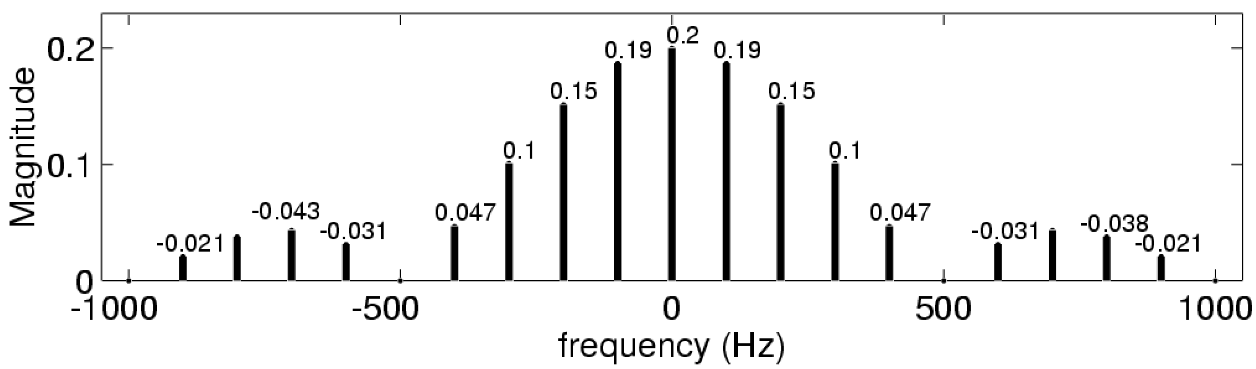
PROBLEM 3.9 (more):

(e) $t_c = T_0/10$

Part (e): Spectrum for $t_c = T_0/10$



(f) When $t_c = T_0/10$ the high frequency components (above 500 Hz) are <u>relatively</u> larger, and there are more high freq. components. For example,

$$\left|\frac{X_7}{X_0}\right| = \frac{0.045}{0.5} = 0.09 \text{ for } t_c = T_0/4\text{, but is } 0.215 \text{ for } t_c = T_0/10$$

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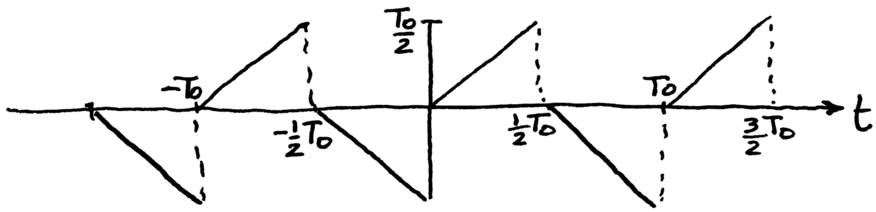
PROBLEM 3.10:

Half-wave symmetry: $x(t+T_0/2) = -x(t)$

(a) $x(t) = t$ for $0 \le t < T_0/2$



(b)
$$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j0}dt$$

Change of variables $u = t - T_0/2$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)dt$$

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)} du$$

$= -x(u)$ because of half-wave symmetry

$$= \frac{1}{T_0}\int_0^{T_0/2} x(t)dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)du = \underline{\underline{0}}$$

(c) If k is even, then $k = 2\ell$

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)e^{-j(2\pi/T_0)2\ell t}dt + \frac{1}{T_0}\int_{T_0/2}^{T_0} x(t)e^{-j(2\pi/T_0)2\ell t}dt$$

Make the same change of variables: $u = t - T_0/2$.

$$a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)e^{-j(2\pi/T_0)2\ell t}dt + \frac{1}{T_0}\int_0^{T_0/2} \underbrace{x(u+T_0/2)}_{=-x(u)} e^{-j(2\pi/T_0)2\ell(u+T_0/2)}du$$

$$e^{-j(2\pi/T_0)2\ell u}e^{-j(2\pi/T_0)2\ell T_0/2} = e^{-j(2\pi/T_0)2\ell u}\underbrace{e^{-j2\pi\ell}}_{=1}$$

$$\Rightarrow a_k = \frac{1}{T_0}\int_0^{T_0/2} x(t)e^{-j(2\pi/T_0)2\ell t}dt - \frac{1}{T_0}\int_0^{T_0/2} x(u)e^{-j(2\pi/T_0)2\ell u}du$$

$\Rightarrow a_k = 0$ for k even.

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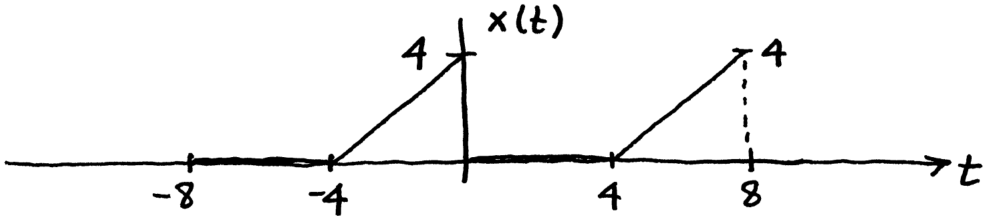
PROBLEM 3.11:

$$a_k = \frac{1}{8}\int_{-4}^{0}(4+t)\,e^{-j(2\pi/8)kt}\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,e^{-j(2\pi/T)kt}\,dt$$

(a) $T = 8$ secs & $x(t) = \begin{cases} (4+t) & \text{for } -4 \le t \le 0 \\ 0 & \text{for } 0 < t < 4 \end{cases}$

(b)



(c) $$a_0 = \frac{1}{8}\int_{-4}^{0}(t+4)\,e^{-j0}\,dt = \frac{1}{8}\left(\frac{t^2}{2}+4t\right)\Big|_{-4}^{0}$$

$$= 0 - \frac{1}{8}\left(\frac{16}{2}-16\right) = 1$$

or, $a_0 = \frac{1}{8} \times$ Area-in-one-period

$$= \frac{1}{8}\times\left(\frac{1}{2}\times 4\times 4\right) = 1$$

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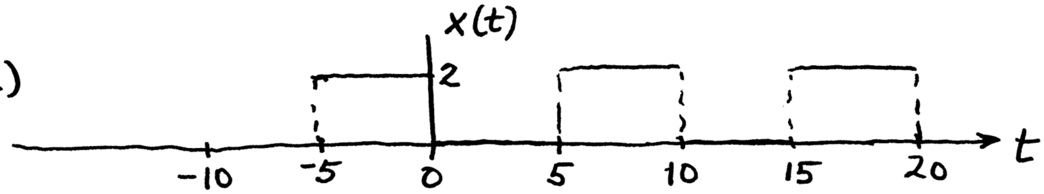
PROBLEM 3.12:

$$x(t) = \begin{cases} 0 & 0 \le t \le 5 \\ 2 & 5 < t \le 10 \end{cases} \qquad T_0 = 10 \text{ secs.}$$

(a)



(b) $a_0 = \frac{1}{T_0} \times \text{Area} = \frac{1}{10} \times (5 \times 2) = 1$

(c) $$a_1 = \frac{1}{10} \int_5^{10} 2 e^{-j(2\pi/10)t} dt$$

$$= \frac{1}{5} \left. \frac{e^{-j\pi t/5}}{(-j\pi/5)} \right|_5^{10} = \frac{e^{-j2\pi} - e^{-j\pi}}{-j\pi} = \frac{1-(-1)}{-j\pi} = \frac{2j}{\pi}$$

(d) $$y(t) = 1 + x(t) = 1 + \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 kt}$$

$$= (1 + a_0) + \sum_{k \neq 0} a_k e^{j\omega_0 kt}$$

$$\Longrightarrow b_0 = 1 + a_0 \quad \text{and} \quad b_1 = a_1$$

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PROBLEM 3.13:

$$x(t) = \sin(10t)\left(\sum_{k=-3}^{3} \frac{1}{1+jk} e^{jkt}\right)$$

$$= \left(\frac{1}{2j}e^{j10t} - \frac{1}{2j}e^{-j10t}\right)\left(\sum_{k=-3}^{3} \frac{1}{1+jk} e^{jkt}\right)$$

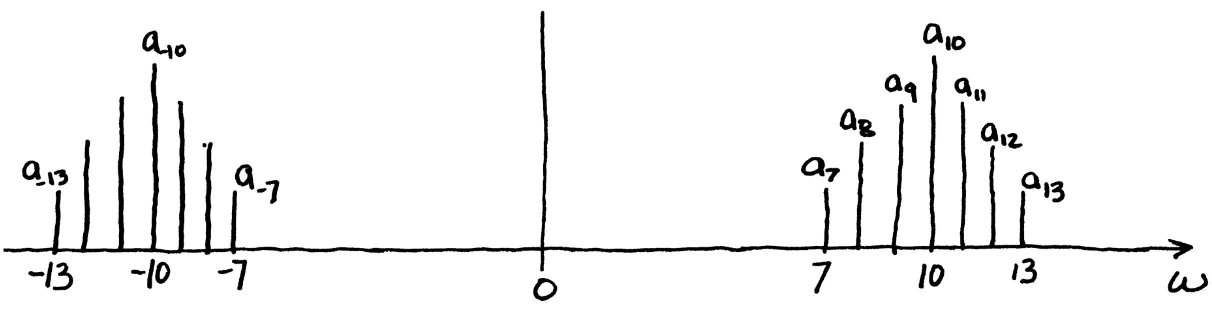
$$= \sum_{k=-3}^{3} \frac{1}{2j-2k} e^{j(k+10)t} + \sum_{k=-3}^{3} \frac{1}{-2j+2k} e^{j(k-10)t}$$

change index $\ell = k+10$ change index $r = k-10$

$$x(t) = \sum_{\ell=7}^{13} \frac{1}{2j-2\ell+20} e^{j\ell t} + \sum_{r=-13}^{-7} \frac{1}{-2j+2r+20} e^{jrt}$$

Note: $\omega_0 = 1$ rad/s for this signal. The sums above contain the 7^{th} through 13^{th} harmonics.

$$a_k = \begin{cases} \dfrac{1}{2j-2k+20} & k = 7, 8, 9, 10, 11, 12, 13 \\ \dfrac{1}{-2j+2k+20} & k = -7, -8, -9, -10, -11, -12, -13 \\ 0 & \text{elsewhere} \end{cases}$$



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PROBLEM 3.14:

(a) $y(t) = Ax(t) = A \cdot \sum_k a_k e^{jk\omega_0 t}$

$$= \sum_k (Aa_k) e^{jk\omega_0 t} \implies \underline{b_k = Aa_k}$$

(b) $y(t) = x(t - t_d)$

$$= \sum_k a_k e^{j\omega_0 k(t - t_d)}$$

$$= \sum_k \left(a_k e^{-j\omega_0 k t_d}\right) e^{j\omega_0 k t}$$

$$\implies \underline{b_k = a_k e^{-jk\omega_0 t_d}}$$

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PROBLEM 3.15:

(a)

$$a_k = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} 1\, e^{-j(2\pi/T_0)kt}\,dt$$

$$= \frac{1}{T_0}\; \frac{e^{-j(2\pi/T_0)kt}}{(-jk)(2\pi/T_0)}\Bigg|_{-T_0/4}^{T_0/4}$$

$$= \frac{e^{-j(2\pi/T_0)kT_0/4} - e^{+j(2\pi/T_0)kT_0/4}}{-j2\pi k}$$

$$a_k = \frac{e^{j\pi k/2} - e^{-j\pi k/2}}{j2\pi k} = \frac{\sin(\pi k/2)}{\pi k}$$

Note: $a_0 = 1/2$

(b) $y(t) = 2x(t - T_0/2)$

If $y(t) = \sum_k b_k e^{jk\omega_0 t}$ & $x(t) = \sum_k a_k e^{jk\omega_0 t}$

Then $b_k = 2a_k e^{-jk\omega_0 T_0/2}$

Since $\omega_0 = 2\pi/T_0$ the exponent is $-jk\pi$

$$\Rightarrow \quad b_k = \left(2e^{-j\pi k}\right)a_k = \left(2(-1)^k\right)a_k$$

$$\therefore \quad b_k = 2(-1)^k \frac{\sin(\pi k/2)}{\pi k}$$

PROBLEM 3.16:

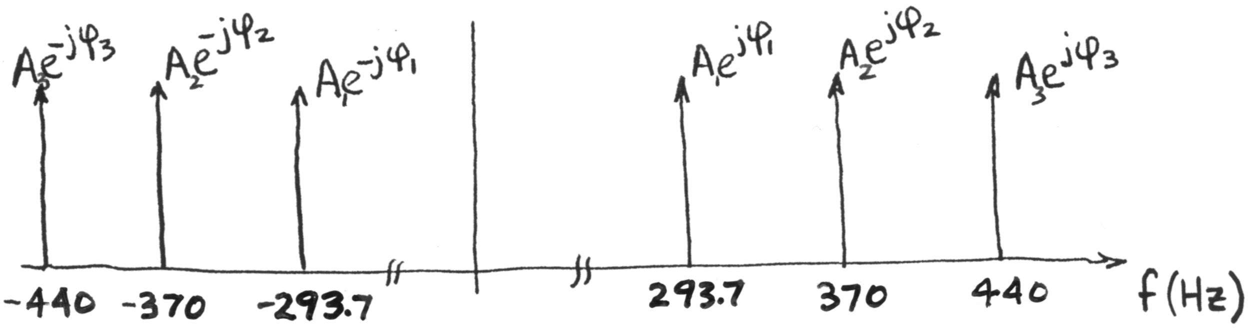
(a) The formula is $(440)\,2^{k/12}$ where k is the number of keys above or below A-440.

key	k	freq (Hz)
C	−9	261.6
C#	−8	277.2
D	−7	293.7
E^b	−6	311.1
E	−5	329.6
F	−4	349.2

key	k	freq (Hz)
F#	−3	370.0
G	−2	392.0
G#	−1	415.3
A	0	440
B^b	1	466.2
B	2	493.9
C	3	523.3

(b) $f = (440)\,2^{(n-49)/12}$ ← key number for A-440

(c) D-F#-A has three frequencies



SPECTRUM of TYPICAL
D-major CHORD

Assuming amplitude of each note is the same

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PROBLEM 3.17:

(a) $$\omega_i(t) = \frac{d}{dt}\psi(t) = 2\alpha t + \beta$$

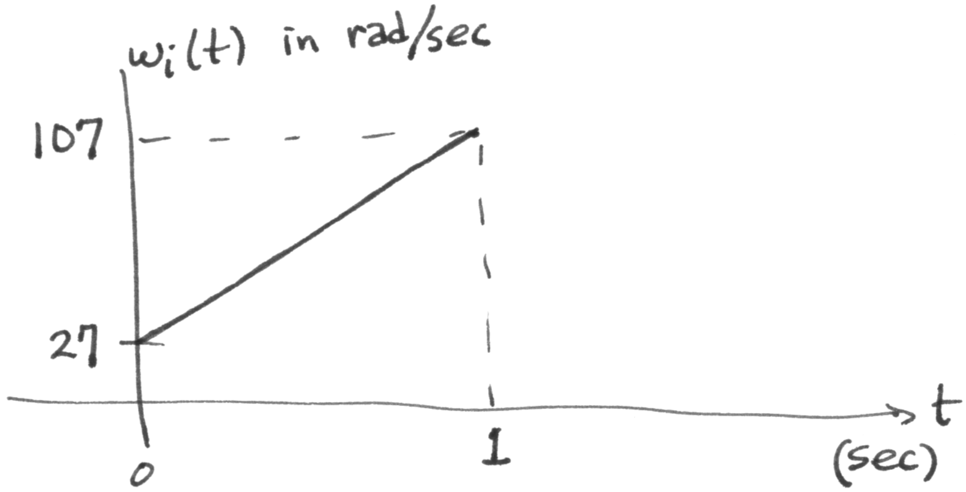
$$\omega_1 = \omega_i(t)\Big|_{t=0} = \beta$$ BEGINNING FREQ

$$\omega_2 = \omega_i(t)\Big|_{t=T_2} = 2\alpha T_2 + \beta$$ ENDING FREQ

(b) $$\psi(t) = 40t^2 + 27t + 13$$

$$\omega_i(t) = \frac{d}{dt}\psi = 80t + 27$$

(c)



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PROBLEM 3.18:

(a) $\omega_i(t) = 2\alpha t + \beta$

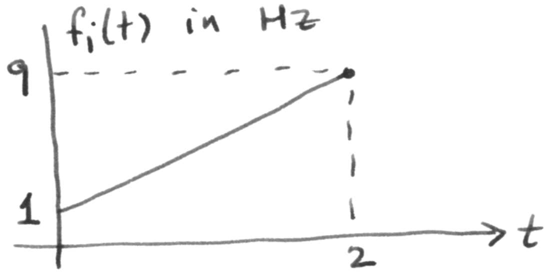
$$\text{SLOPE} = 2\alpha = \frac{\omega_2 - \omega_1}{T_2} \Rightarrow \alpha = \frac{2\pi(9) - 2\pi}{4} = 4\pi$$

$$\text{INTERCEPT} = \beta = \omega_1 = 2\pi$$

$$\therefore \psi(t) = 4\pi t^2 + 2\pi t + \varphi$$

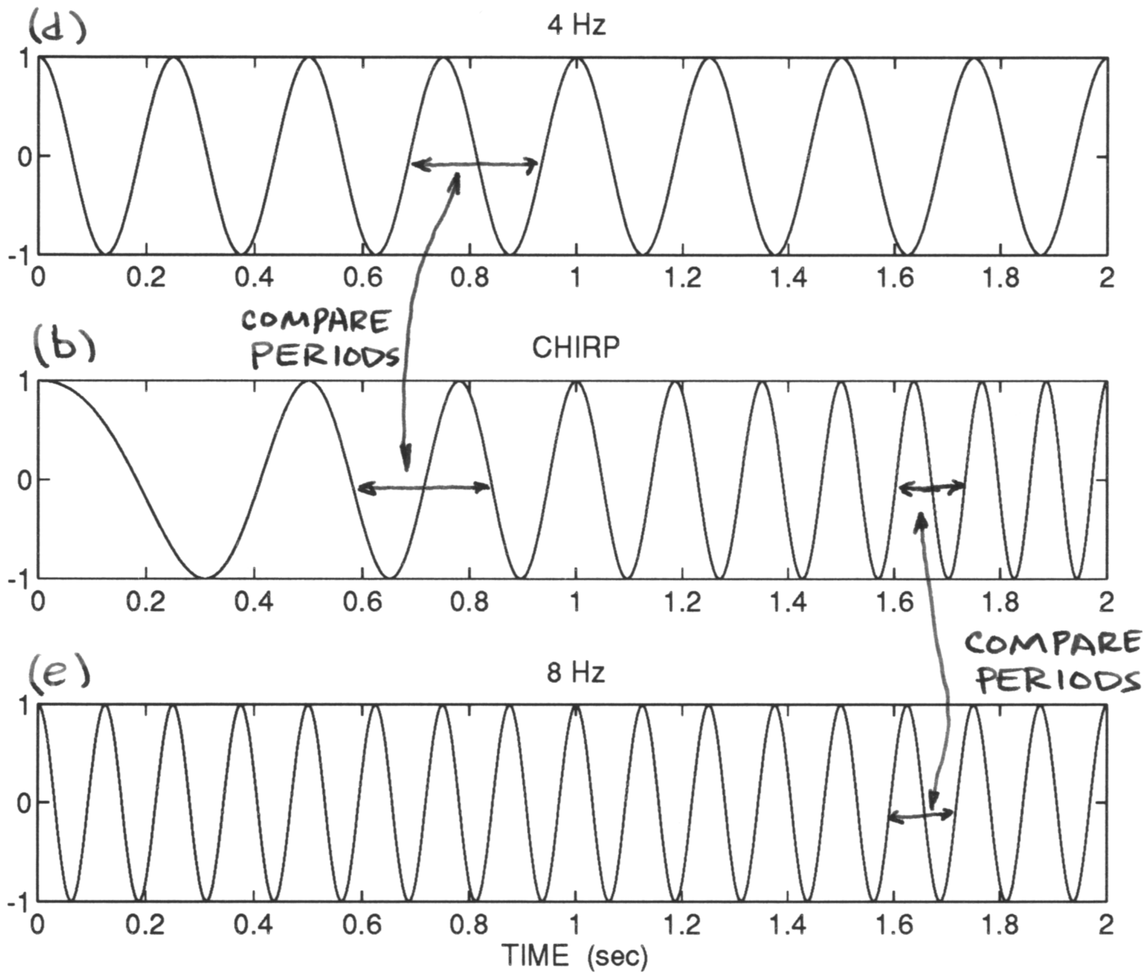
ANY φ WILL WORK

(c)



at $t = 3/4$, $f_i = 4$ Hz

at $t = 7/4$, $f_i = 8$ Hz.



(f) In the time interval $1.6 \le t \le 2$ sec, the 8Hz sinusoid looks a lot like the chirp, so the $f_i(t)$ is obviously close to 8Hz in the interval $1.6 \rightarrow 2.0$

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Characterize each time signal:

(a) 6 periods from $t=-2$ to $t=+3$ $\Rightarrow T = \frac{5}{6}$ sec
DC value = 2 $\quad t_m > 0 \Rightarrow \varphi < 0$

(b) 3 periods from $t=0$ to $t=2$ $\Rightarrow T = \frac{2}{3}$ sec
DC value = 0 $\quad \varphi = \pi$

(c) 6 periods from $t=-3$ to $t=2$ $\Rightarrow T = \frac{5}{6}$ sec
DC value = 2 $\quad t_m < 0 \Rightarrow \varphi > 0$

(d) Period $\approx 3\frac{1}{3} = \frac{10}{3}$ secs. Two frequencies
DC value = 0

(e) 2 periods from $t=-2$ to $t=3$ $\Rightarrow T = 2.5$ secs
Two frequencies. DC value = 0

(1) $\omega_0 = 2\pi(1.2) \Rightarrow T = 1/1.2 = 5/6$ sec
$\varphi = 0.5\pi > 0 \quad$ DC = 2

(2) $\omega_0 = 2\pi(0.3)$ because 0.3 divides 0.6 & 1.5
$\Rightarrow T = 1/0.3 = 10/3$ secs. $\quad$ DC = 0

(3) Like (1). $T = 5/6$ sec. $\quad$ DC = 2
But $\varphi = -0.25\pi < 0$

(4) $\omega_0 = 2\pi(0.4) \Rightarrow T = 1/0.4 = 2.5$ secs
DC = 0

(5) $\varphi = \pi \quad T = 1/1.5 = \frac{2}{3}$ sec

Thus, the match is

(a) $\leftrightarrow$ (3) $\quad$ (c) $\leftrightarrow$ (1) $\quad$ (e) $\leftrightarrow$ (4)

(b) $\leftrightarrow$ (5) $\quad$ (d) $\leftrightarrow$ (2)

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