

2.4 Error Analysis for Iterative Methods

1) Order of convergence of a sequence.

Def: Suppose $\{p_n\}_{n=1}^{\infty}$ converges to p (with $p_n \neq p \forall n$).

If $\lambda > 0$, $\alpha > 0$ exist with

$$\lim_{n \rightarrow \infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha} = \lambda$$

then $\{p_n\}_{n=1}^{\infty}$ converges to p of order α .

λ is called asymptotic error constant.

(i) If $\alpha = 1$, then $0 < \lambda \leq 1$ and the sequence is linearly convergent.

(ii) If $\alpha = 2$, the sequence is quadratically convergent.

Ex: 1) The sequence $\frac{1}{n^2}$ converges to zero linearly. Indeed

$$\frac{\left| \frac{1}{(n+1)^2} - 0 \right|}{\frac{1}{n} |n|^\alpha} = \frac{\frac{1}{(n+1)^2}}{\frac{1}{n}} = \frac{n^{2\alpha}}{(n+1)^2} = \frac{n^{2\alpha}}{n^2 + 2n + 1} \xrightarrow{\alpha=1} 1$$