

Ex: Consider the fixed point equation

$$x = \frac{2 - e^x + x}{3}$$

(a) Determine an interval $[a, b]$ which contains a solution of the fixed point problem

$$\text{Let } f(x) = x - \frac{2 - e^x + x}{3}$$

$$f(0) = -\frac{2 - 1 + 0}{3} = -\frac{1}{3} < 0$$

$$f(1) = 1 - \frac{2 - e + 1}{3} = 1 - \frac{3 - e}{3} = 1 - 1 + \frac{e}{3} > 0$$

Thus, the interval $[0, 1]$ contains a solution

(b) Show that the iteration

$$p_{n+1} = \frac{2 - e^{p_n} + p_n}{3}$$

will converge for every p_0 in $[0, 1]$

Solution: Consider $p_{n+1} = g(p_n)$

where $g(x) = \frac{2 - e^x + x}{3}$