

$$= 0.423 + 0.4306 + 0.0456 - 0.006 = 0.893$$

Note: The function actually is  $f(x) = \tan x$   
So  $f(0.73) = 0.895$ . Thus, the error is 0.002.

Similarly, we can define backward differences. Consider the points

$$x_n, x_{n-1}, \dots, x_0$$

Def: The first backward difference at  $x_i$  is defined as

$$\nabla f(x_i) = f(x_i) - f(x_{i-1})$$

Note:  $\nabla f(x_i) = \Delta f(x_{i-1})$

Def: The  $k$ th backward difference at  $x_i$  is defined as

$$\nabla^k f(x_i) = \nabla^{k-1} f(x_i) - \nabla^{k-1} f(x_{i-1})$$

Newton's backward-difference formula

$$P_n(x_n + sh) = P_n(s) = f(x_n) + s \nabla f(x_n) + \frac{s(s+1)}{2!} \nabla^2 f(x_n) + \frac{s(s+1)(s+2)}{3!} \nabla^3 f(x_n) \\ + \dots + \frac{s(s+1)\dots(s+n-1)}{n!} \nabla^n f(x_n)$$