1) Iteration (a) diverges
2) Iteration (d) can't start with that p_0
3) Iteration (e) converges to the root fastest.

Question: How do we know what fixed-point problem will produce a sequence that reliably and rapidly converge to a solution?

Theorem 2.3 (Fixed-Point Theorem)
Let 1) $g \in C[a,b]$
2) $a \le g(x) \le b$ for all $x \in [a,b]$
3) $g'(x)$ exists on (a,b)
4) there is a constant k : $0 < k < 1$
such that
$$|g'(x)| \le k \quad \text{for all } x \in (a,b)$$
Then, for any number p_0 in $[a,b]$ the sequence
$$p_n = g(p_{n-1}) \quad n \ge 1$$
converges to the unique fixed point $p \in [a,b]$.

Corollary 2.4 If g satisfies the hypotheses of Theorem 2.3, then the bounds for the error involved in using p_n to approximate p are given by