Ex. Let $f(x) = x^3$. The first forward difference at the points $x_0 = 1$ $x_1 = 2$ is

$$\Delta f(x_0) = f(2) - f(1) = 2^3 - 1^3 = 7$$

Def. The <u>second forward difference</u> $\Delta^2 f(x_i)$ $\Delta^2 f(x_i)$ is defined as follows

$$\Delta^2 f(x_i) = \Delta f(x_{i+1}) - \Delta f(x_i)$$

Consequently, the divided difference is

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{f[x_{i+1}, x_{i+2}] - f[x_{i+1}, x_i]}{x_{i+2} - x_i} =$$

$$= \frac{1}{2h}\left[\frac{\Delta f(x_{i+1})}{h} - \frac{\Delta f(x_i)}{h}\right]$$

$$= \frac{1}{2h^2}\Delta^2 f(x_i)$$

Def. The <u>$(k+1)^{st}$ forward difference</u> $\Delta^{k+1} f(x_i)$ is defined as follows

$$\Delta^{k+1} f(x_i) = \Delta^k f(x_{i+1}) - \Delta^k f(x_i)$$

In general,

$$f[x_i, \ldots, x_{i+k}] = \frac{1}{k!\,h^k}\Delta^k f(x_i)$$