

(b) set up a composite Simpson's rule with $n=6$ to approximate the integral

$$\int_a^b f(x) dx = \frac{h}{3} [f(a) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(b)]$$

$$\begin{array}{cccccc} | & | & | & | & | & | \\ \frac{\pi}{6} & \frac{2\pi}{6} & \frac{\pi}{2} & \frac{4\pi}{6} & \frac{5\pi}{6} & \pi \end{array}$$

$$h = \frac{\pi}{6}$$

$$\begin{aligned} \int_0^{\pi} x^2 \cos x dx &\approx \frac{\pi}{6 \cdot 3} \left[0 + 4\left(\frac{\pi}{6}\right)^2 \cos \frac{\pi}{6} + 2\left(\frac{\pi}{3}\right)^2 \cos \frac{\pi}{3} \right. \\ &\quad + 4 \cos \left(\frac{\pi}{2}\right)^2 \cos \frac{\pi}{2} + 2\left(\frac{2\pi}{3}\right)^2 \cos \left(\frac{2\pi}{3}\right) + 4\left(\frac{5\pi}{6}\right)^2 \cos \frac{5\pi}{6} \\ &\quad \left. + (\pi^2) \cos \pi \right] \end{aligned}$$

Ex. Approximate the integral

$$\int_1^{\frac{3}{2}} x^2 \ln x dx$$

using Gaussian quadrature with $n=4$.

Solution: To use Gaussian quadrature we need to change the variables so

$$\int_1^{\frac{3}{2}} \rightarrow \int_{-1}^1$$