PROBLEM 13.1:

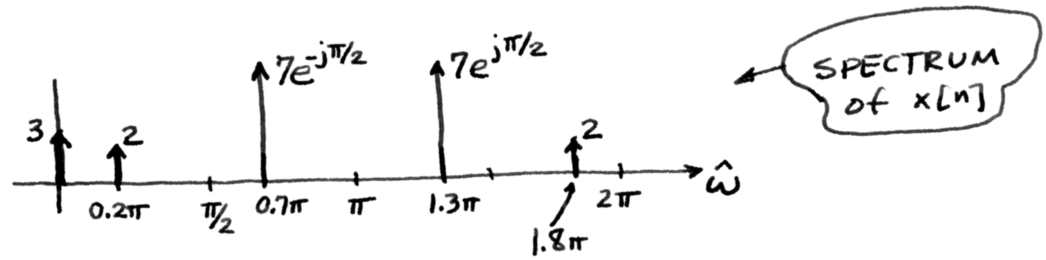
$$x[n] = 3 + 2e^{j0.2\pi n} + 2e^{-j0.2\pi n} - 7j\,e^{j0.7\pi n} + 7j\,e^{-j0.7\pi n}$$

$$-7j = 7e^{-j\pi/2} \qquad 7j = 7e^{j\pi/2}$$

(a)

$\hat{\omega} = -0.2\pi$ is same as $\hat{\omega} = 2\pi - 0.2\pi = 1.8\pi$

$\hat{\omega} = -0.7\pi \longrightarrow \hat{\omega} = 2\pi - 0.7\pi = 1.3\pi$

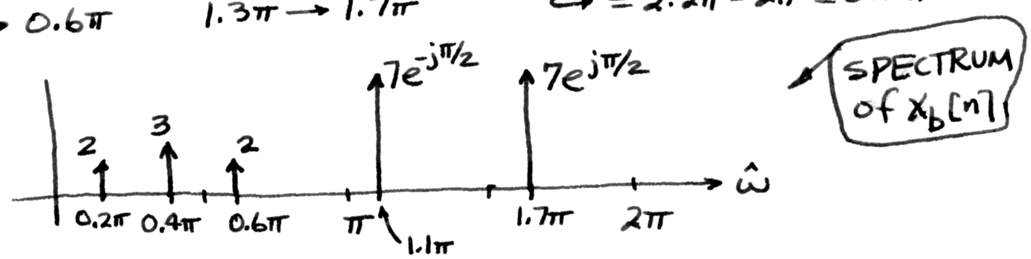


(b) $x_b[n] = x[n]e^{j0.4\pi n}$

All frequencies will be increased by 0.4π

$0 \rightarrow 0.4\pi \qquad 0.7\pi \rightarrow 1.1\pi \qquad 1.8\pi \rightarrow 2.2\pi$

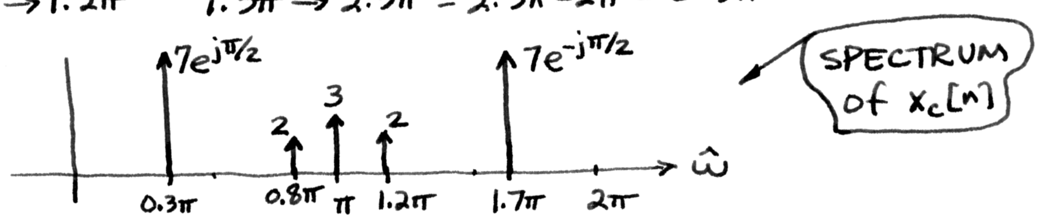
$0.2\pi \rightarrow 0.6\pi \qquad 1.3\pi \rightarrow 1.7\pi \qquad = 2.2\pi - 2\pi = 0.2\pi$



(c) $(-1)^n = e^{j\pi n}$, so all frequencies increase by π.

$0 \rightarrow \pi \qquad 0.7\pi \rightarrow 1.7\pi \qquad 1.8\pi \rightarrow 2.8\pi \rightarrow 0.8\pi$

$0.2\pi \rightarrow 1.2\pi \qquad 1.3\pi \rightarrow 2.3\pi = 2.3\pi - 2\pi = 0.3\pi$



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PROBLEM 13.2:

(a) The rectangular window is $w_R(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$

Using the Fourier transform table for a delayed pulse (with delay $= T/2$), we get

$$W_R(j\omega) = e^{-j\omega T/2} \frac{\sin(\omega T/2)}{\omega/2}$$

(b) $w_H(t) = w_R(t)\left[0.54 - 0.23 e^{j(2\pi/T)t} - 0.23 e^{-j(2\pi/T)t}\right]$

Using the frequency-shifting property of the F.T.

$$W_H(j\omega) = 0.54 W_R(j\omega) - 0.23 W_R(j(\omega - \tfrac{2\pi}{T}))$$
$$- 0.23 W_R(j(\omega + \tfrac{2\pi}{T}))$$

(c) The width of the mainlobe of $W_R(j\omega)$ is $4\pi/T$.

For the Hamming window, $W_H(j\omega)$ is the sum of 3 sinc functions that are displaced from one another by $2\pi/T$:

$$W_H(j\omega) = e^{-j\omega T/2}\left\{0.54 \frac{\sin(\omega T/2)}{\omega/2} - 0.23 e^{+j(\frac{2\pi}{T})\frac{T}{2}} \frac{\sin(\omega\frac{T}{2} - \pi)}{(\omega - 2\pi/T)/2}\right.$$
$$\left. - 0.23 e^{-j\pi} \frac{\sin(\omega\frac{T}{2} + \pi)}{(\omega + 2\pi/T)/2}\right\}$$

Notice that the exponentials are $e^{\pm j\pi} = -1$, so they cancel the minus signs in front of -0.23. Thus the sinc functions add approximately. The result is shown in the figures below.

PROBLEM 13.3:

Since $X_c(j\omega) = 0$ for $|\omega| \geq 2\pi(1000)$, the minimum sampling rate to avoid aliasing is $f_s > 2000$ Hz.

The effective spacing between DFT frequencies is $\Delta\hat{\omega} = 2\pi/N$ which corresponds to f_s/N Hz.

Thus, a 5 Hz spacing requires

$$\frac{f_s}{N} < 5 \qquad \text{or} \qquad f_s < 5N$$

Putting the two requirements together, we get:

$$2000 < f_s < 5N$$

$$\Longrightarrow \quad N > 400.$$

Since $N = 2^{\nu}$, the minimum N is $\boxed{N = 512}$

Then we can use $f_s < 5N$ to get a maximum f_s:

$$\underbrace{2000}_{f_s^{min}} < f_s < 5(512) = \underbrace{2560}_{f_s^{max}} \text{ Hz.}$$

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PROBLEM 13.4:

(a) $$X_0[k] = \sum_{n=0}^{9} x_0[n] e^{-j(2\pi/10)nk} = x_0[0] e^{-j0} = 1$$

only non-zero for $n=0$

(b) $$X_1[k] = \sum_{n=0}^{9} 1\, e^{-j(2\pi/10)nk}$$

$$= \begin{cases} 10 & \text{if } k=0 \\ \dfrac{1-e^{-j(2\pi/10)10k}}{1-e^{-j2\pi/10\,k}} = \dfrac{1-1}{\text{denom}} = 0 & \text{if } k=1,2,\ldots 9 \end{cases}$$

(c) $$X_2[k] = \sum_{n=0}^{9} x_2[n] e^{-j(2\pi/10)nk} = e^{-j(2\pi/10)4k}$$

non-zero only for $n=4$

$$X_2[k] = e^{-j(4\pi/5)k} \quad \text{for } k=0,1,2,\ldots 9$$

(d) $$X_3[k] = \sum_{n=0}^{9} e^{j2\pi n/5} e^{-j(2\pi/10)nk}$$

$$= \sum_{n=0}^{9} e^{j(2\pi/10)(2n-nk)}$$

$$= \begin{cases} 10 & \longleftarrow \text{ when } k=2 \\ \dfrac{1-e^{j(2\pi/10)10(2-k)}}{1-e^{j(2\pi/10)(2-k)}} = \dfrac{1-1}{\text{denom}} = 0 & \text{when } k\neq 2 \end{cases}$$

$$X_3[k] = \begin{cases} 10 & \text{for } k=2 \\ 0 & \text{for } k=0,1,3,4,5,6,7,8,9 \end{cases}$$

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PROBLEM 13.5:

(a) $x_a[n] = \frac{1}{10}\sum_{k=0}^{9} X_a[k] e^{j(2\pi/10)nk} = \frac{1}{10} X_a[0] e^{j0} = \frac{1}{10}$

non-zero only for $k=0$

(b) $x_b[n] = \frac{1}{10}\sum_{k=0}^{9} 1\, e^{j(2\pi/10)nk} = \frac{1}{10}\cdot\frac{1-e^{j(2\pi/10)10n}}{1-e^{j(2\pi/10)n}}$

Numerator is always zero because $1-e^{j2\pi n} = 1-1=0$

The denominator is non-zero when $n=1,2,\ldots 9$.

When $n=0$, we can evaluate the sum directly:

$$x_b[0] = \frac{1}{10}\sum_{k=0}^{9} 1\, e^{j0} = \frac{1}{10}(10) = 1.$$

$$x_b[n] = 0 \quad \text{for } n=1,2,3,\ldots 9$$

(c) $x_c[n] = \frac{1}{10}\sum_{k=0}^{9} X_c[k] e^{j(2\pi/10)nk}$

non-zero at $k=3,7$

$$= \frac{1}{10}\left(e^{j(2\pi/10)3n} + e^{j(2\pi/10)7n}\right)$$

$e^{j2\pi(7/10)} = e^{-j2\pi(3/10)}$

$$x_c[n] = \frac{1}{5}\cos\left(2\pi\left(\tfrac{3}{10}\right)n\right)$$

(d) $X_d[k] = \cos(2\pi k/5) = \frac{1}{2}e^{j(2\pi/10)2k} + \frac{1}{2}e^{-j(2\pi/10)2k}$

Use orthogonality property (Prob 9.5) to get the IDFT of a complex exponential.

The IDFT of $\frac{1}{2}e^{-j(2\pi/10)m_0 k}$ is all zeros except for $n=m_0$ where the answer is $\frac{1}{20}(10)=\frac{1}{2}$

Also $\frac{1}{2}e^{j(2\pi/10)2k} = \frac{1}{2}e^{-j(2\pi/10)8k}$

$$\Rightarrow x_d[n] = \begin{cases} \frac{1}{2} & \text{for } n=2,8 \\ 0 & \text{for } n=0,1,3,4,5,6,7,9 \end{cases}$$

PROBLEM 13.6:

(a) $$Y_0[k] = e^{j0} + e^{-j(2\pi/12)k} + e^{-j(2\pi/12)2k} + e^{-j(2\pi/12)3k}$$

$$= e^{-j(2\pi/12)1.5k}\left(e^{+j(2\pi/12)1.5k} + e^{+j(2\pi/12)\frac{1}{2}k} + e^{-j(2\pi/12)\frac{1}{2}k} + e^{-j(2\pi/12)1.5k}\right)$$

$$Y_0[k] = e^{-j(2\pi/12)1.5k}\left(2\cos\left(\left(\tfrac{2\pi}{12}\right)1.5k\right) + 2\cos\left(\left(\tfrac{2\pi}{12}\right)\tfrac{1}{2}k\right)\right)$$

(b) $y_1[n] = 1$ for $n = 2\ell$, $\ell = 0, 1, \ldots 5$

$$Y_1[k] = \sum_{\ell=0}^{5} 1\, e^{-j(2\pi/12)(2\ell)k} = \sum_{\ell=0}^{5} e^{-j(2\pi/6)\ell k}$$

The sum is equal to 6 when $k =$ multiple of 6. This can be proven with the <u>orthogonality</u> <u>property</u> of Prob 9.5. Thus

$$Y_1[k] = \begin{cases} 6 & \text{for } k = 0, 6 \\ 0 & \text{for } k = 1,2,3,4,5,\ 7,8,9,10,11 \end{cases}$$

(c) $$Y_2[k] = \sum_{n=0}^{4} e^{j2\pi n/5} e^{-j(2\pi/12)nk}$$

Let $\theta = 2\pi(\frac{1}{5} - k/12)$. Then the sum is $\sum_{n=0}^{4} e^{j\theta n}$

$$\sum_{n=0}^{4} e^{j\theta n} = \frac{1 - e^{j5\theta}}{1 - e^{j\theta}} = \frac{e^{j\frac{5}{2}\theta}}{e^{j\theta/2}} \cdot \frac{e^{-j\frac{5}{2}\theta} - e^{j\frac{5}{2}\theta}}{e^{-j\theta/2} - e^{j\theta/2}}$$

$$= e^{j4\theta}\,\frac{\sin(5\theta/2)}{\sin(\frac{1}{2}\theta)}$$

$$\Rightarrow Y_2[k] = e^{j2\pi(\frac{4}{5} - k/3)}\,\frac{\sin\left(5\pi\left(\frac{1}{5} - \frac{k}{12}\right)\right)}{\sin\left(\pi\left(\frac{1}{5} - \frac{k}{12}\right)\right)}$$

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PROBLEM 13.7:

The 8-point running sum filter has coefficients

$$\{b_k\} = \{1, 1, \ldots, 1\} \quad \text{(8 coefficients)}$$

Its frequency response is:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{7} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{7} e^{-j\hat{\omega}k}$$

After some manipulation $H(e^{j\hat{\omega}})$ can be expressed as a "Dirichlet" form:

$$H(e^{j\hat{\omega}}) = e^{-j7\hat{\omega}/2} \frac{\sin(4\hat{\omega})}{\sin(\frac{1}{2}\hat{\omega})}$$

Since the numerator, $\sin(4\hat{\omega})$, is <u>ZERO</u> for $\hat{\omega} = \pi/4$, $\pi/2$ and $3\pi/4$, the spectrum components at $\hat{\omega}_1 = 0.25\pi$, $\hat{\omega}_2 = 0.5\pi$ and $\hat{\omega}_3 = 0.75\pi$ will all be removed. The output $y[n]$ will be the DC value of $x[n]$ multiplied by $H(e^{j0})$ which is

$$H(e^{j0}) = \sum_{k=0}^{7} e^{j0} = \sum_{k=0}^{7} 1 = 8$$

$$\Rightarrow \quad y[n] = 8X_0 = 8(3) = 24$$

PROBLEM 13.7 (more):

(d) In this case, there are two "transient" regions $0 \le n < 11$ and $20 \le n < 31$.

For $n<0$ and $n \ge 31$ the output is zero.

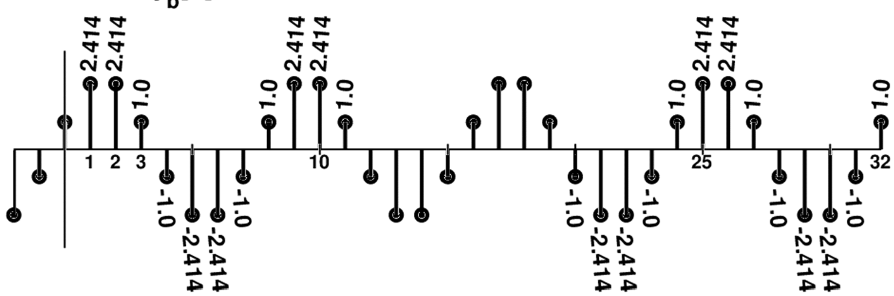
In the "steady-state" region $11 \le n < 20$, we get the same answer as part (b).

$$y_d[n] = 0 \quad \text{for } n<0 \text{ or } n \ge 31$$

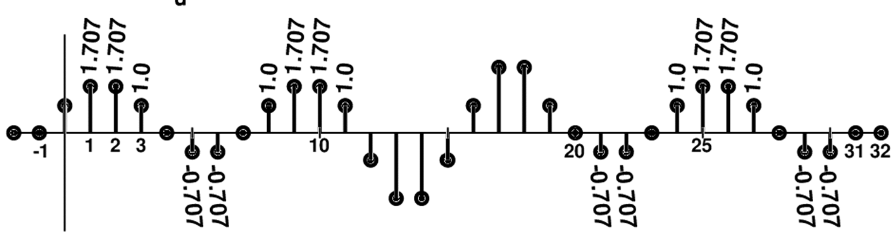
$$y_d[n] = 2.613\, e^{j(\pi n/4 - 3\pi/8)} \quad \text{for } 11 \le n < 20$$

The real & imaginary parts of $y_d[n]$ are plotted.

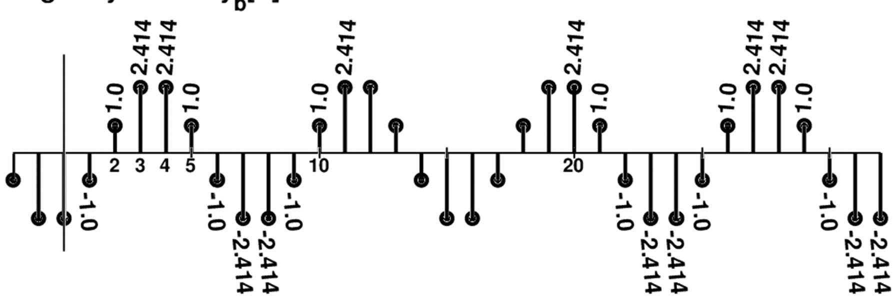
Real Part of $y_b[n]$



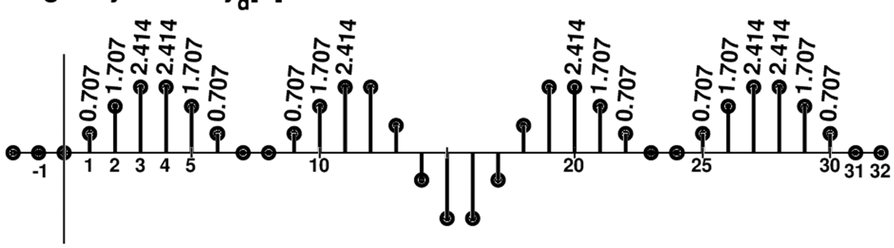
Real Part of $y_d[n]$



Imaginary Part of $y_b[n]$



Imaginary Part of $y_d[n]$



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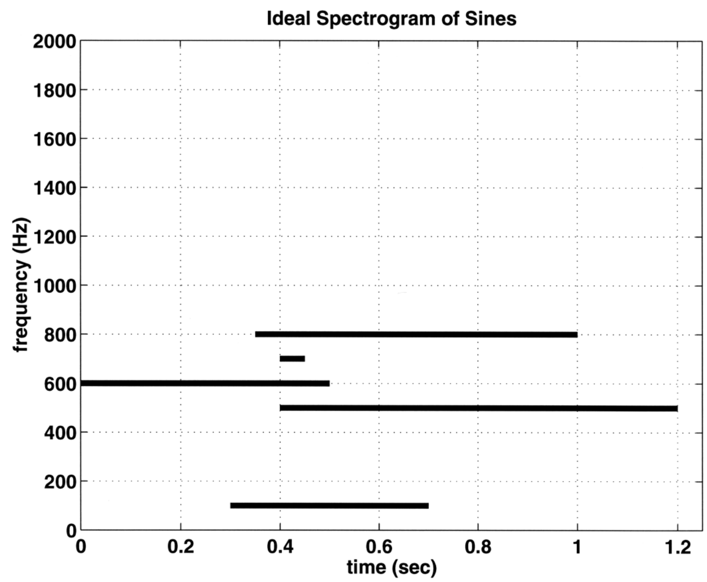
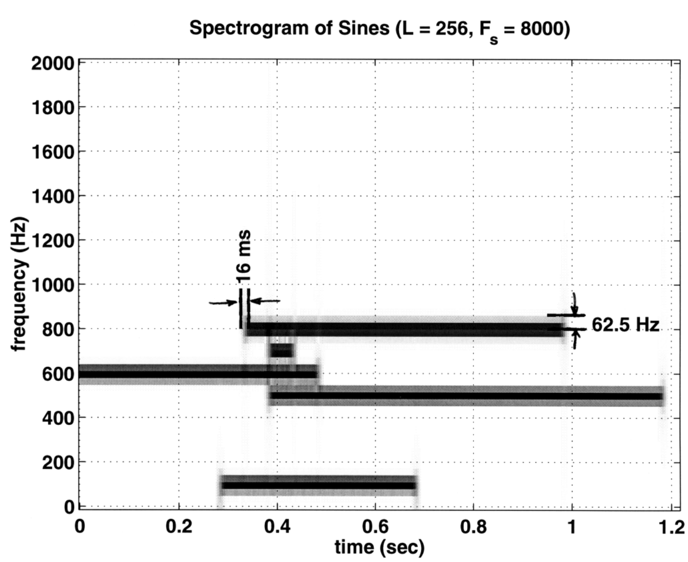
PROBLEM 13.8:

(a) No amplitude information is given, so we assume all sinusoids have the same amplitude. The figure below shows the starting and ending times for each frequency component.



(b) The spectrogram below was computed via MATLAB. Its frequency resolution is: $f_{RES} = \frac{2}{L} f_s = 62.5\,\text{Hz}$ which is marked on the plot.
Likewise, the time resolution $t_{RES} = L/2f_s = 16\,\text{ms}$ is indicated on the plot.



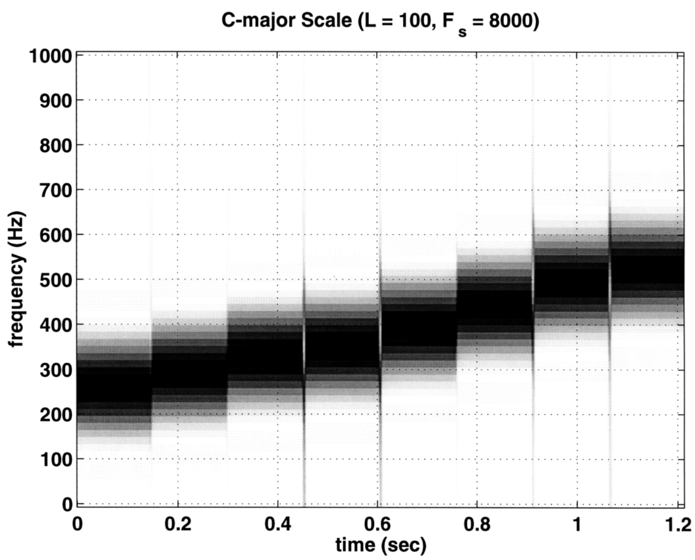
PROBLEM 13.9:

The spectrograms below were created via MATLAB.

For $L=100$ the resolutions are:

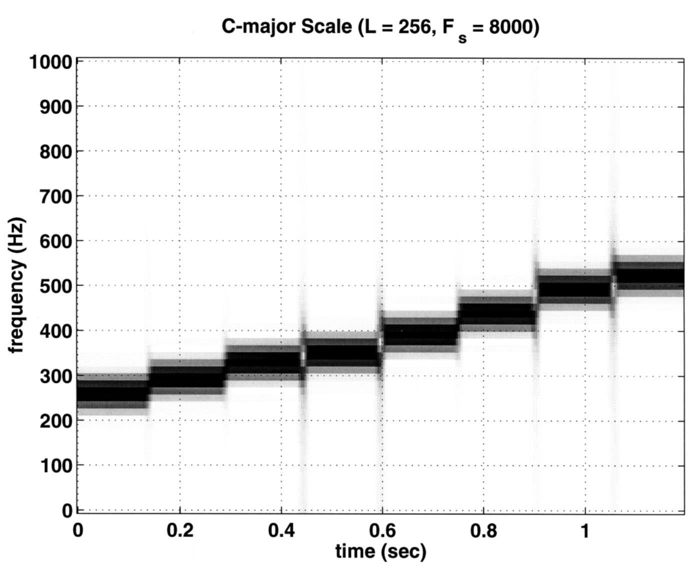
$$f_{RES} = \frac{2}{L} f_s = \frac{2}{100} \times 8000 = 160\text{ Hz}$$

$$t_{RES} = L/2f_s = \frac{1}{160}\text{ sec}$$



When $L=256$,

$$f_{RES} = \frac{2}{256} \times 8000 = 62.5\text{ Hz} \quad \& \quad t_{RES} = \frac{256}{2 \times 8000} = 16\text{ msec}$$



This spectrogram is shown for comparison.

PROBLEM 13.10:

It appears that each note has the same duration, and there are 7 notes during the time interval $0 \rightarrow 1.05\,s$. Thus the duration of a single note is $\frac{1.05}{7} = 0.15$ sec.

Note: the original version of the figure was labeled incorrectly. The time axis should <u>not</u> be <u>milli</u>sec.

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PROBLEM 13.11:

Here are some "measurements" in Fig 13-23:

$\{650, 610, 650, 610, 650, 480, 580, 520, 440\}$ Hz

The calculated frequencies are:

Key-Number	Key	Frequency (Hz)
56	E	659.25
55	D#	622.25
56	E	659.25
55	D#	622.25
56	E	659.25
51	B	493.88
54	D	587.33
52	C	523.25
49	A	440

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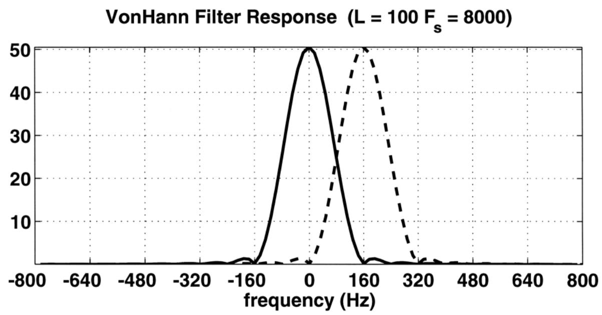
PROBLEM 13.12:

In order to give the frequency resolution in Hz and the time resolution in sec, we need to know the sampling frequency f_s. Let's assume $f_s = 8000$ Hz.

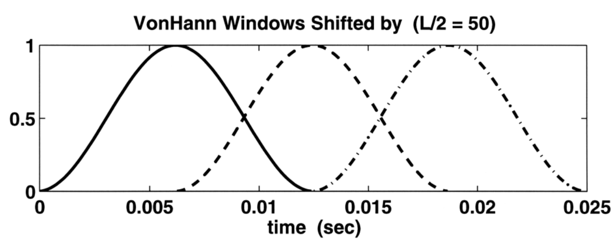
(a) $f_{RES} = \frac{2}{L} f_s = \frac{2}{100} \times 8000 = 160$ Hz

The figure below shows the frequency response of two 100-pt vonHann filters separated by 160 Hz. They overlap at the $\frac{1}{2}$-Amplitude point.



(b) $t_{RES} = \frac{L}{2} T_s = \frac{L}{2}(1/f_s) = \frac{100}{2}(1/8000) = \frac{1}{160} = 0.00625$ sec

The figure below shows vonHann windows shifted by 0.00625 sec. They overlap at the $\frac{1}{2}$-Amplitude point.



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PROBLEM 13.13:

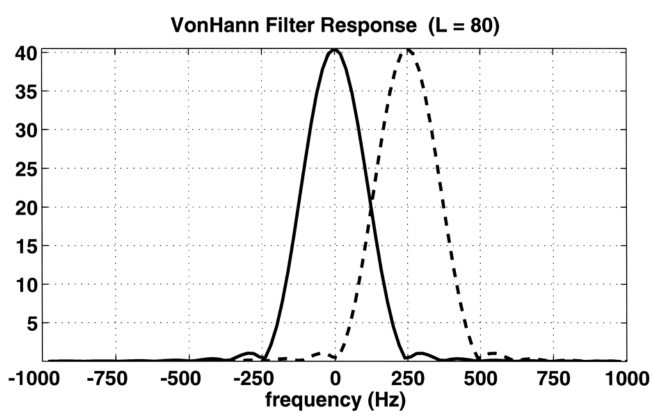
(a) $f_{RES} = \frac{2}{L} f_s \Rightarrow L = 2(f_s/f_{RES})$

$$L = 2(10{,}000/250) = 2(40) = 80 \leftarrow \text{minimum value}$$

(b) The vonHann filter has the following impulse response:

$$h[n] = \begin{cases} \frac{1}{2}(1-\cos(2\pi n/80)) & \text{for } n = 0, 1, \ldots 79 \\ 0 & \leftarrow \text{elsewhere} \end{cases}$$

The frequency response $H(e^{j\hat{\omega}})$ is shown below. The frequency axis has been scaled to Hz. Only the central portion near $\hat{\omega} = 0$ is shown.



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