PROBLEM 5.3:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) MAKE A TABLE:

n	<0	0	1	2	3	4	5	6	7	≥ 8
$x[n]$	0	1	2	3	2	1	1	1	1	1
$y[n]$	0	2	1	2	-1	2	3	1	1	1 ......

$y[0] = 2x[0] - 3x[-1] + 2x[-2] = 2(1) = 2$

$y[1] = 2x[1] - 3x[0] + 2x[-1] = 2(2) - 3(1) = 1$

$y[2] = 2x[2] - 3x[1] + 2x[0] = 2(3) - 3(2) + 2(1) = 2$

$y[3] = 2(2) - 3(3) + 2(2) = -1$

$y[4] = 2(1) - 3(2) + 2(3) = 2$

$y[5] = 2(1) - 3(1) + 2(2) = 3$

$y[6] = 2(1) - 3(1) + 2(1) = 1$

$y[7] = 2(1) - 3(1) + 2(1) = 1$

$y[8] = 2(1) - 3(1) + 2(1) = 1$

(c) Impulse Response

$h[0] = 2(1) - 3(0) + 2(0) = 2$

$h[1] = 2(0) - 3(1) + 2(0) = -3$

$h[2] = 2(0) - 3(0) + 2(0) = 2$

Notice $h[n]$ just "reads out" the filter coefficients:

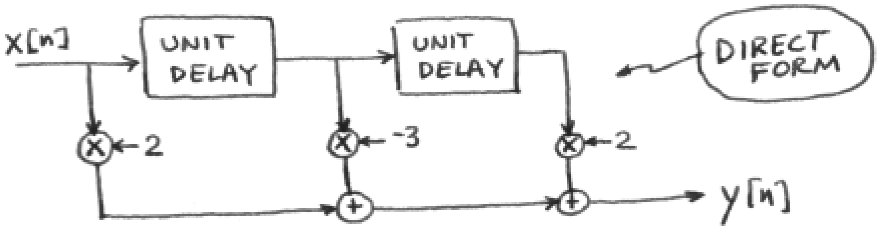
i.e., $h[n] = b_n$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

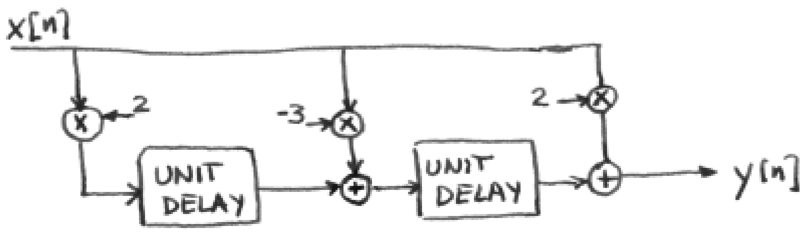
PROBLEM 5.4:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2].$$

(a)



(b) Transposed Form



PROBLEM 5.6:

Plots for parts (a), (b) and (c) are below.

(d) This general solution will also apply to part (c).

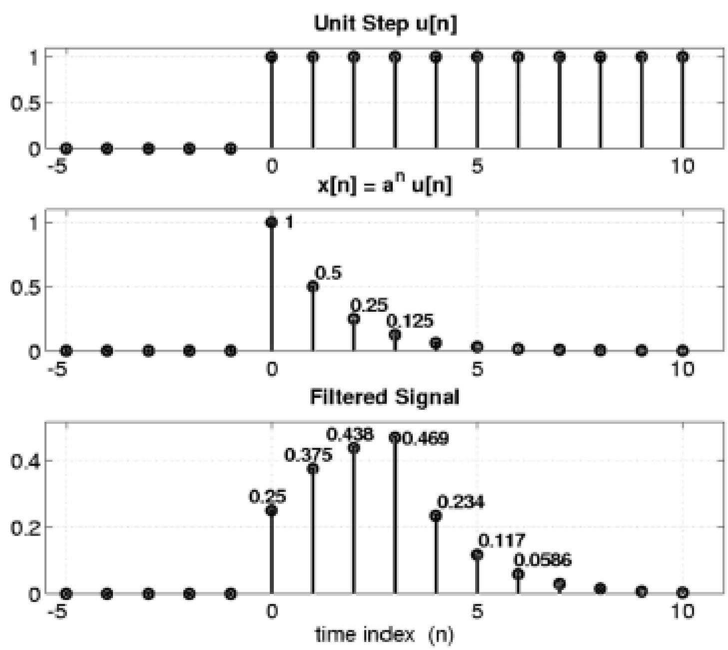
$$x[n] = a^n u[n] \qquad y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k} u[n-k]$$

There are 3 cases.

1. $n<0$. $\Rightarrow y[n]=0$ because $u[n-k]$ is always zero

2. $0 \le n \le L-1$
$$y[n] = \frac{1}{L}\sum_{k=0}^{n} a^{n-k} u[n-k] = \frac{a^n}{L}\sum_{k=0}^{n} a^{-k}$$
$$y[n] = \frac{a^n}{L}\left(\frac{1-a^{-n-1}}{1-a^{-1}}\right) = \frac{1}{L}\left(\frac{a^{n+1}-1}{a-1}\right)$$

3. $n \ge L$
$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k} \cancelto{1}{u[n-k]} = \frac{a^n}{L}\sum_{k=0}^{L-1} a^{-k}$$
$$= \frac{a^n}{L}\left(\frac{1-a^{-L}}{1-a^{-1}}\right) = \frac{a^n}{L}\left(\frac{a^L-1}{a^L-a^{L-1}}\right) \quad \text{for } n \ge L.$$



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 5.7:

$$h[n] = 3\delta[n] + 7\delta[n-1] + 13\delta[n-2] + 9\delta[n-3] + 5\delta[n-4]$$

The impulse response contains the values of the filter coefficients because

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

Thus,

$$b_0 = 3, \quad b_1 = 7, \quad b_2 = 13, \quad b_3 = 9, \quad b_4 = 5$$

PROBLEM 5.9:

Linearity ?

(a) YES.

Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$

$x_1[n] \to y_1[n]$
$x_2[n] \to y_2[n]$

$\Rightarrow y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n])\cos(0.2\pi n)$

$= \alpha_1 \underbrace{x_1[n]\cos(0.2\pi n)}_{y_1[n]} + \alpha_2 \underbrace{x_2[n]\cos(0.2\pi n)}_{y_2[n]}$

(b) YES.

$y[n] = (\alpha_1 x_1[n] - \alpha_2 x_2[n]) - (\alpha_1 x_1[n-1] + \alpha_2 x_2[n-1])$

$= \alpha_1 (\underbrace{x_1[n] - x_1[n-1]}_{y_1[n]}) + \alpha_2 (\underbrace{x_2[n] - x_2[n-1]}_{y_2[n]})$

(c) NO.

Let $x_1[n] = \delta[n]$ and $x_2[n] = -2\delta[n]$.

$\hookrightarrow y_1[n] = \delta[n]$ $\quad\hookrightarrow y_2[n] = |x_2[n]| = 2\delta[n]$

Let $x[n] = x_1[n] + x_2[n] = \delta[n] - 2\delta[n] = -\delta[n]$

$\hookrightarrow y[n] = |x[n]| = \delta[n] \longleftrightarrow y_1[n] + y_2[n] = \delta[n] + 2\delta[n] = 3\delta[n]$

NOT EQUAL!

(d) NO! if $B \neq 0$

if $x_1[n] \to y_1[n]$, test $2x_1[n] \to 2y_1[n]$.

$A(2x_1[n]) + B = 2(Ax_1[n] + B) - B \neq 2y_1[n]$

TIME - INVARIANT?

(a) NO!

Let $x[n] = \delta[n]$, then $y[n] = \delta[n]\cos(0.2\pi n) = \delta[n]$

EVAL AT $n=0$

Try $x[n-1] = \delta[n-1]$, then output is

$\delta[n-1]\cos(0.2\pi n) = \cos(0.2\pi)\delta[n-1]$.

But $\cos(0.2\pi)\delta[n-1] \neq y[n-1] = \delta[n-1]$

McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

PROBLEM 5.17:

(a) $h_1[n] = \delta[n] - \delta[n-1]$

$h_2[n] = \delta[n] + \delta[n-2]$

$h_3[n] = \delta[n-1] + \delta[n-2]$

(b) The overall $h[n]$ is the convolution of the $h_i[n]$.

$$h[n] = h_1[n] * h_2[n] * h_3[n]$$

$$h_1[n] * h_2[n] = \left(\delta[n] - \delta[n-1]\right) * \left(\delta[n] + \delta[n-2]\right)$$

$$= \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

Now convolve with $h_3[n]$

$$\begin{array}{cccccc}
1 & -1 & 1 & -1 & & \\
0 & 1 & 1 & & & \\
\hline
0 & 0 & 0 & 0 & & \\
 & 1 & -1 & 1 & -1 & \\
 & & 1 & -1 & 1 & -1 \\
\hline
0 & 1 & 0 & 0 & 0 & -1 \\
n=0 & \uparrow_{n=1} & & & & \uparrow_{n=5}
\end{array}$$

$$\boxed{h[n] = \delta[n-1] - \delta[n-5]}$$

(c) $y[n] = h[n] * x[n]$

$$= \left(\delta[n-1] - \delta[n-5]\right) * x[n]$$

$$y[n] = x[n-1] - x[n-5]$$