First we define linear functions

$$L_0(x) \quad , \quad L_1(x)$$

such that

$$L_0(x_0) = 1 \qquad L_0(x_1) = 0$$
$$L_1(x_0) = 0 \qquad L_1(x_1) = 1$$

namely,

$$L_0(x) = \frac{x - x_1}{x_0 - x_1} \qquad L_1(x) = \frac{x - x_0}{x_1 - x_0}$$

then we define

$$P(x) = L_0(x) f_0 + L_1(x) f_1$$

Thus, $P(x_0) = L_0(x_0) f_0 + L_1(x_0) f_1 = f_0$
$P(x_1) = L_0(x_1) f_0 + L_1(x_1) f_1 = f_1$

Note: $P(x)$ is the unique linear function passing through the points

$$(x_0, f_0) \qquad (x_1, f_1)$$

Now we generalize to $n+1$ points.

Given

$$\begin{array}{cccc} x_0 & x_1 & \cdots & x_n \\ f_0 & f_1 & & f_n \end{array}$$

First, we construct polynomials