$$P_n(x) = f_0 L_0(x) + f_1 L_1(x) + \cdots + f_n L_n(x)$$

where

$$L_k(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_n)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_n)}$$

Let $f(x) = \sqrt{x}$. Write down the Lagrange polynomial of degree 2 which interpolates $f(x)$ in the points $1, 2, 3$

Solution:

x	1	2	3
$f(x)$	1	$\sqrt{2}$	$\sqrt{3}$

$$L_0(x) = \frac{(x-2)(x-3)}{(-1)(-2)} = \frac{1}{2}(x-2)(x-3)$$

$$L_1(x) = \frac{(x-1)(x-3)}{(1)(2-3)} = -(x-1)(x-3)$$

$$L_2(x) = \frac{(x-1)(x-2)}{(2)(2-1)} = \frac{1}{2}(x-1)(x-2)$$

$$P_2(x) = 1 \cdot L_0(x) + \sqrt{2} L_1(x) + \sqrt{3} L_2(x)$$

$$= \frac{1}{2}(x-2)(x-3) - \sqrt{2}(x-1)(x-3) + \frac{\sqrt{3}}{2}(x-1)(x-2)$$