Ex. #6 Find the rate of convergence of

$$\lim_{n\to\infty} \sin\frac{1}{n^2} = 0$$

$$\sin\frac{1}{n^2} \leq \frac{1}{n^2}$$

Thus the rate of convergence is $O(\frac{1}{n^2})$.

Def: Suppose $\lim_{h\to 0} G(h) = 0$ and $\lim_{h\to 0} F(h) = L$

If a positive constant K exists with

$$|F(h) - L| \leq K|G(h)| \text{ for } h\text{-small}$$

then we write

$$F(h) = L + O(G(h))$$

- Usually: $G(h) = h^p$, where $p > 0$.

Ex. #7c Find the rate of convergence of

$$\lim_{h\to 0} \frac{\sinh h - h\cosh h}{h} = 0$$

$$\sinh h \approx h - \frac{h^3}{3!} \qquad \cosh h \approx 1 - \frac{h^2}{2!} + \frac{h^4}{4!}$$

$$\frac{\sinh h - h\cosh h}{h} \approx \frac{h - \frac{h^3}{3!} - h + \frac{h^3}{2!}}{h} = \frac{1}{3}h^2 = O(h$$