$$L_{2,0}(x) = \frac{(x-0.6)(x-0.9)}{(0-0.6)(0-0.9)} \qquad L_{2,0}(0.45) = 0.125$$

$$L_{2,1}(x) = \frac{x(x-0.9)}{(0.6-0)(0.6-0.9)} \qquad L_{2,1}(0.45) = 1.125$$

$$L_{2,2}(x) = \frac{x(x-0.6)}{(0.9-0)(0.9-0.6)} \qquad L_{2,2}(0.45) = -0.25$$

Note $\cos 0 = 1$ $\cos 0.6 \approx 0.825$ $\cos 0.9 \approx 0.6216$

$$P(x) = L_{2,0}(x) + (\cos 0.6)L_{2,1}(x) + (\cos 0.9)L_{2,2}(x)$$

$$P(0.45) = 0.125 + (\cos 0.6)\cdot 1.125 + (\cos 0.9)(-0.25)$$

$$P(0.45) = 0.898100074\overline{7}$$

Actual value $= 0.9004471024$

Error $= \cos(0.45) - P(0.45) = 0.0023470276$

Error bound:

$$E(x;f) = \frac{f'''(\xi(x))}{3!}(x-0)(x-0.6)(x-0.9)$$

$$|E(x;f)| \le \frac{|\sin\xi(x)|}{6}|x(x-0.6)(x-0.9)| \le \frac{\sin 0.9}{6}0.05704 = 0.0001493$$

$f'(x) = -\sin x$, $f''(x) = -\cos x$ $f'''(x) = \sin x$

Note: $\sin x$ is increasing in $[0, 0.9]$

Let $g(x) = x(x-0.6)(x-0.9)$ $g'(x) = 3(x^2 - x + 0.18)$

$x_1 = 0.7645751311$ $x_2 = 0.2354248689$

$g(x_1) = 0.05704051584$

$g(x_2) = -0.01704051584$