$$P(x) = L_{3,0}(x) + 2L_{3,1}(x) + 3L_{3,2}(x) + 4L_{3,3}(x)$$

3) The interpolation error

Theorem 3.3 Suppose $x_0, x_1, \dots, x_n$ are $n+1$ distinct numbers in the interval $[a,b]$ and $f(x)$ has $n+1$ continuous derivatives. Then for each x in $[a,b]$, a number $\xi(x)$ in (a,b) exists with

$$f(x) = P(x) + \overbrace{\frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\dots(x-x_n)}^{E_n(x;f)}$$

where $P(x)$ is the n^{th} Lagrange interpolating polynomial.

Def: $E_n(x;f) = f(x) - P(x) = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}(x-x_0)\dots(x-x_n)$ - error

Ex #1/119 For the function

$$f(x) = \cos x$$

let $x_0 = 0$, $x_1 = 0.6$, $x_2 = 0.9$. Construct the interpolation polynomial of degree at most two to approximate $f(0.45)$ and find the actual error. Use Theorem 3.3 to find the error bound for the error