2.2 Fixed-Point Iteration

1) Fixed point of a function

Def: A number p is called a <u>fixed point</u> for a given function $g(x)$ if

$$g(p) = p$$

<u>In this section we consider the problems:</u>

1) Finding solutions to fixed-point problems
2) Connection between the fixed-point problem and root finding problems

<u>Root-finding problems are equivalent to fixed-point problems:</u>

Given $f(p) = 0$ we can define $g(x)$ with a fixed-point at p in a number of ways

Ex: a) $g(x) = x - f(x)$
b) $g(x) = x + 3f(x)$

Ex: a) $x^2 + x + 1 = 0 \to x = -x^2 - 1$
b) $\ln x + 1 = 0 \to x + \ln x + 1 = x$

Conversely, if p is a fixed point of $g(x)$ that is $g(p) = p$, then the function

$$f(x) = x - g(x)$$

has a zero at p.

Ex: a) $x = \ln x \to x - \ln x = 0$
b) $x = \sqrt{x+1} \to x - \sqrt{x+1} = 0$