

Ex. Use Theorem 2.2 to show that the function $g(x) = \frac{1}{2}e^{-x}$ has a unique fixed point in $[0, 1]$.

Solution: $g(x)$ - continuous on $[0, 1]$
 $0 \leq g(x) \leq \frac{1}{2} \leq 1$

$$g'(x) = -\frac{1}{2}e^{-x} \Rightarrow |g'(x)| \leq \frac{1}{2}$$

Ex Note in Example 1 that $g(x)$ has a unique fixed point in $[2, 4]$ but

Thus, $g(2) = -2$ $g(4) = 10$
it is not true

$$2 \leq g(x) \leq 4$$

Also

$$g'(x) = 2x$$

and is clearly not smaller than one

Thus, the conditions of Theorem 2.2 is sufficient but not necessary.