$$f'''(x) = -[\cos(\ln x) - \sin(\ln x)] \cdot \frac{1}{x^3}$$
$$+2[\sin(\ln x) + \cos(\ln x)] \cdot \frac{1}{x^3}$$
$$= [3\sin(\ln x) + \cos(\ln x)] \cdot \frac{1}{x^3}$$

To bound the 3rd derivative we find the fourth

$$f^{IV}(x) = [3\cos(\ln x) - \sin(\ln x)] \cdot \frac{1}{x^4}$$
$$-3[3\sin(\ln x) + \cos(\ln x)] \cdot \frac{1}{x^4}$$
$$= -10\sin(\ln x) \cdot \frac{1}{x^4} < 0$$

$\Rightarrow f'''(x)$ is decreasing on $[2, 2.6]$.

$$|f'''(x)| \leq [3\sin(\ln 2) + \cos(\ln 2)] \cdot \frac{1}{2^3} \leq 0.335765$$

Next, we need the maximum of

$$g(x) = (x-2)(x-2.4)(x-2.6)$$

$$g'(x) = (x-2.4)(x-2.6) + (x-2)(x-2.6) + (x-2)(x-2.4)$$
$$= x^2 - 5x + 6.24 + x^2 - 4.6x + 5.2 + x^2 - 4.4x + 4.8$$
$$= 3x^2 - 14x + 16.24 = 0$$
$$x_1 = 2.5 \qquad x_2 = 2.157$$