

PROBLEM 6.4:



$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

$$(a) \quad \mathcal{X}(\hat{\omega}) = \sum_{k=0}^2 b_k e^{-j\hat{\omega}k} = 2 - 3e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}$$

Simplify with symmetry:

$$\mathcal{X}(\hat{\omega}) = e^{-j\hat{\omega}} \{ 2e^{j\hat{\omega}} - 3 + 2e^{-j\hat{\omega}} \}$$

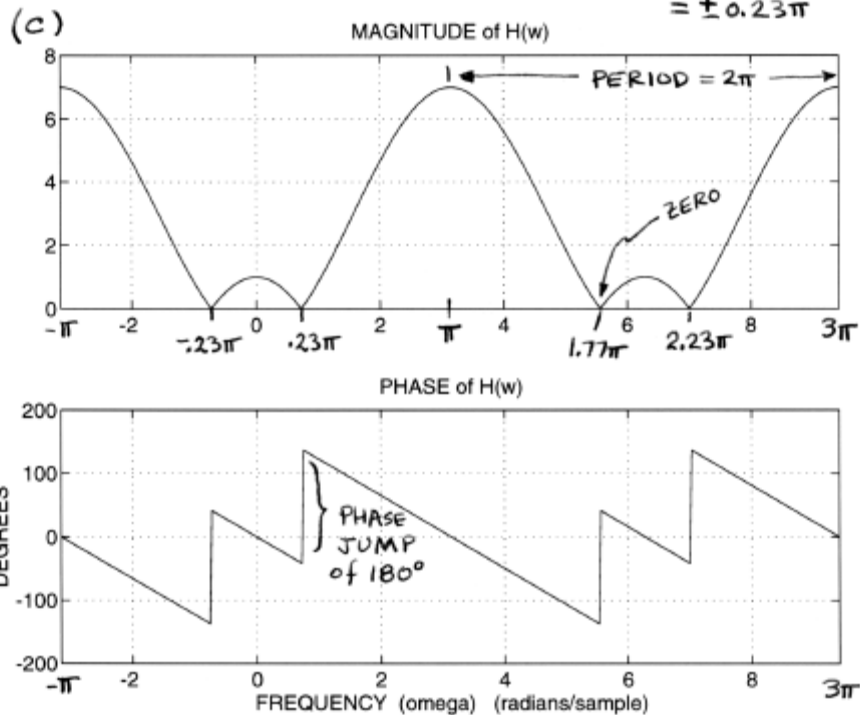
$$= (4 \cos \hat{\omega} - 3) e^{-j\hat{\omega}}$$

PHASE

THIS TERM IS MAG, EXCEPT FOR A POSSIBLE MINUS SIGN

$$(b) \quad \cos \hat{\omega} \text{ has PERIOD} = 2\pi \Rightarrow \mathcal{X}(\hat{\omega}) \text{ has period} = 2\pi$$

$$(d) \quad \text{OUTPUT IS ZERO WHEN } \mathcal{X}(\hat{\omega}) = 0 \\ \Rightarrow \text{SOLVE } 4 \cos \hat{\omega} - 3 = 0 \Rightarrow \hat{\omega} = \cos^{-1}(3/4) \\ = \pm 0.23\pi$$



PROBLEM 6.4 (more):

$$(e) \quad \mathcal{X}(\hat{\omega}) \text{ at } \hat{\omega} = \frac{\pi}{13} \text{ is } \mathcal{X}\left(\frac{\pi}{13}\right) = 0.8838 e^{-j0.077\pi}$$

Since the freq. response alters mag & phase of the input, we can get output via:

$$x[n] = \sin\left(\frac{\pi}{13}n\right) = \cos\left(\frac{\pi}{13}n - \frac{\pi}{2}\right).$$

$$\begin{aligned} \Rightarrow y[n] &= 0.8838 \cos\left(\frac{\pi}{13}n - 0.5\pi - 0.077\pi\right) \\ &= 0.8838 \cos\left(\frac{\pi}{13}n - 0.577\pi\right) \quad \text{for all } n \end{aligned}$$

Another approach which uses linearity:

$$\sin\left(\frac{\pi}{13}n\right) = \frac{1}{2j} e^{j\pi n/13} - \frac{1}{2j} e^{-j\pi n/13}$$

$$\Rightarrow y[n] = \mathcal{X}\left(\frac{\pi}{13}\right) \frac{1}{2j} e^{j\pi n/13} - \mathcal{X}\left(-\frac{\pi}{13}\right) \frac{1}{2j} e^{-j\pi n/13}$$

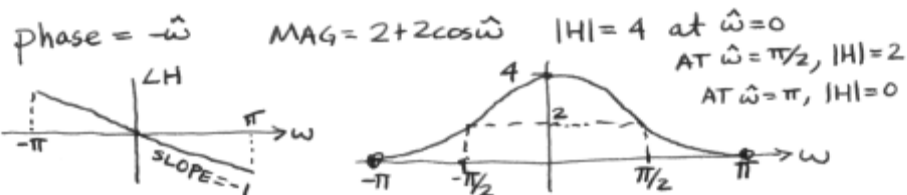
PROBLEM 6.5:

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(a) use filter coeffs: $\{b_k\} = \{1, 2, 1\}$

$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

(b) $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$



(c) $x[n] = 10 + 4\cos(\frac{\pi}{2}n + \frac{\pi}{4})$

$$= 10 + 2e^{j\pi/4}e^{j\pi/2n} + 2e^{-j\pi/4}e^{-j\pi/2n}$$

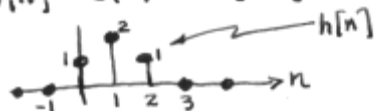
$$y[n] = 10\mathcal{H}(0) + \mathcal{H}(\pi/2)2e^{j\pi/4}e^{j\pi/2n} + 2\mathcal{H}(-\pi/2)e^{-j\pi/4}e^{-j\pi/2n}$$

$$\mathcal{H}(0) = 4e^{j0} \quad \mathcal{H}(\pi/2) = e^{-j\pi/2}(2) \quad \mathcal{H}(-\pi/2) = 2e^{j\pi/2}$$

$$\Rightarrow y[n] = 40 + 4e^{-j\pi/2}e^{j\pi/4}e^{j\pi/2n} + 4e^{j\pi/2}e^{-j\pi/4}e^{-j\pi/2n}$$

$$= 40 + 8\cos(\frac{\pi}{2}n - \frac{\pi}{4})$$

(d) $x[n] = \delta[n] \Rightarrow y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$



(e) $x[n] = u[n]$

$$y[n] = u[n] + 2u[n-1] + u[n-2]$$

$$y[n] = 0 \text{ for } n < 0$$

$$y[0] = u[0] + 2u[-1] + u[-2] = 1 + 0 + 0 = 1$$

$$y[1] = u[1] + 2u[0] + u[-1] = 1 + 2 + 0 = 3$$

$$y[2] = u[2] + 2u[1] + u[0] = 1 + 2 + 1 = 4$$

$$y[n] = 4 \text{ for } n \geq 2.$$

PROBLEM 6.7:

(a) $H(e^{j\hat{\omega}}) = 1 + 2e^{-j3\hat{\omega}}$

Solution: Use the fact that the frequency response for $\delta[n - n_0]$ is $H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}$.

$$h[n] = \delta[n] + 2\delta[n - 3]$$

(b) $H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}} \cos(\hat{\omega})$

Solution: Use the inverse Euler formula to write the frequency response in terms of complex exponentials.

$$H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}} \cos(\hat{\omega}) = e^{-j3\hat{\omega}} (e^{j\hat{\omega}} + e^{-j\hat{\omega}})$$

$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$$

$$\Rightarrow h[n] = \delta[n - 2] + \delta[n - 4]$$

(c) $H(e^{j\hat{\omega}}) = e^{-j4.5\hat{\omega}} \frac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)}$

Solution: Use the fact that the frequency response for an L -point running sum filter is:

$$H_L(e^{j\hat{\omega}}) = e^{-j\hat{\omega}(L-1)/2} \frac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

Thus, we see that $L/2 = 5$, or $L = 10$, and we can rewrite the frequency response as

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}(10-1)/2} \frac{\sin(10\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

because $(L - 1)/2$ is equal to 4.5 when $L = 10$. Having made these identifications in the formula for $H(e^{j\hat{\omega}})$, we get the impulse response of the 10-point running-sum filter:

$$\begin{aligned} h[n] &= u[n] - u[n - 10] \\ &= \delta[n] + \delta[n - 1] + \delta[n - 2] + \delta[n - 3] + \delta[n - 4] \dots \\ &\quad + \delta[n - 5] + \delta[n - 6] + \delta[n - 7] + \delta[n - 8] + \delta[n - 9] \end{aligned}$$

PROBLEM 6.9:

$$\begin{aligned}
 \text{(a)} \quad \mathcal{H}(\hat{\omega}) &= (1 - e^{-j\hat{\omega}}) \left(1 - 2(0.5) \cos \frac{\pi}{6} e^{-j\hat{\omega}} + (0.5)^2 e^{-j2\hat{\omega}} \right) \\
 &= (1 - e^{-j\hat{\omega}}) \left(1 - \frac{\sqrt{3}}{2} e^{-j\hat{\omega}} + \frac{1}{4} e^{-j2\hat{\omega}} \right) \\
 &= 1 - \underbrace{\frac{1}{2}(\sqrt{3}+2)}_{-1.866} e^{-j\hat{\omega}} + \underbrace{\left(\frac{1}{4} + \frac{\sqrt{3}}{2}\right)}_{1.116} e^{-j2\hat{\omega}} - \frac{1}{4} e^{-j3\hat{\omega}}
 \end{aligned}$$

Difference Equation:

$$y[n] = x[n] - 1.866x[n-1] + 1.116x[n-2] - \frac{1}{4}x[n-3]$$

(b) when $x[n] = \delta[n]$, $y[n] = h[n]$ impulse response

$$h[n] = \delta[n] - 1.866\delta[n-1] + 1.116\delta[n-2] - \frac{1}{4}\delta[n-3]$$

(c) Find $\hat{\omega}$ where $\mathcal{H}(\hat{\omega}) = 0$

The only frequency is $\hat{\omega} = 0$, because then the factor $(1 - e^{-j\hat{\omega}}) = 0$. The other two factors in $\mathcal{H}(\hat{\omega})$ are never zero for $-\pi \leq \hat{\omega} \leq \pi$.

PROBLEM 6.13:

$$\begin{aligned}
 (a) \quad y[n] &= y_3[n] = x_3[n-1] + x_3[n-2] && \boxed{x_3[n] = y_2[n]} \\
 &= y_2[n-1] + y_2[n-2] \\
 &= (x_2[n-1] + x_2[n-3]) + (x_2[n-2] + x_2[n-4])
 \end{aligned}$$

Now replace $x_2[n]$ with $y_1[n]$

$$\begin{aligned}
 y[n] &= y_1[n-1] + y_1[n-2] + y_1[n-3] + y_1[n-4] \\
 &= (x_1[n-1] - x_1[n-2]) + (x_1[n-2] - x_1[n-3]) \\
 &\quad + (x_1[n-3] - x_1[n-4]) + (x_1[n-4] - x_1[n-5])
 \end{aligned}$$

$$y[n] = x_1[n-1] - x_1[n-5] \quad \boxed{x_1[n] = x[n]}$$

$$y[n] = x[n-1] - x[n-5]$$

(b) Same thing as part (a) but use $\mathcal{H}_i(\hat{\omega})$

$$\left. \begin{aligned}
 \mathcal{H}_1(\hat{\omega}) &= 1 - e^{-j\hat{\omega}} \\
 \mathcal{H}_2(\hat{\omega}) &= 1 + e^{-j2\hat{\omega}} \\
 \mathcal{H}_3(\hat{\omega}) &= e^{-j\hat{\omega}} + e^{j2\hat{\omega}}
 \end{aligned} \right\} \text{Multiply these together}$$

$$\begin{aligned}
 \mathcal{H}_6(\hat{\omega}) &= \mathcal{H}_1(\hat{\omega})\mathcal{H}_2(\hat{\omega})\mathcal{H}_3(\hat{\omega}) \\
 &= (1 - e^{-j\hat{\omega}})(1 + e^{-j2\hat{\omega}})(e^{-j\hat{\omega}} + e^{j2\hat{\omega}}) \\
 &= (1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}) \\
 &= e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} - e^{-j4\hat{\omega}} \\
 &\quad + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}
 \end{aligned}$$

$$\mathcal{H}_6(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\Rightarrow y[n] = x[n-1] - x[n-5]$$

PROBLEM 6.18:

$$X[n] = 5 + 20 \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) + 10\delta[n-3]$$

Need $\mathcal{H}(0)$
DEPENDS on $\mathcal{H}(\pi/2)$
Need impulse response $h[n]$

$$\begin{aligned} \mathcal{H}(0) &= (1-j)(1-(-j))(1+1) \\ &= (1-j)(1+j)^2 = 2 \cdot 2 = 4 \end{aligned}$$

$$\begin{aligned} \mathcal{H}(\pi/2) &= (1-j e^{-j\pi/2})(1+j e^{-j\pi/2})(1+e^{-j\pi/2}) \\ &= (1-j(-j))(1+j(-j))(1-j) \\ &= (1-1)(1+1)(1-j) = 0 \end{aligned}$$

To find $h[n]$, multiply out $\mathcal{H}(\hat{\omega})$

$$\begin{aligned} \mathcal{H}(\hat{\omega}) &= (1-j e^{-j\hat{\omega}} + j e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}}) \\ &= (1+e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}}) \\ &= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} \end{aligned}$$

$$\Rightarrow h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

Finally,

$$\begin{aligned} y[n] &= 5(4) + 0 + 10h[n-3] \\ &= 20 + 10\delta[n-3] + 10\delta[n-4] + 10\delta[n-5] + 10\delta[n-6] \end{aligned}$$