

### 3) Rate of convergence of sequences and functions

Ex. The sequence

$$\left(\sin\left(\frac{1}{n^2}\right)\right)^2$$

converges to zero. Find the rate of convergence

Solution: Since  $\sin x \leq x$

$$\sin \frac{1}{n^2} \leq \frac{1}{n^2}$$

$$\left(\sin \frac{1}{n^2}\right)^2 \leq \frac{1}{n^4}$$

$\Rightarrow$  The rate of convergence is  $O\left(\frac{1}{n^4}\right)$

Ex. Find the rate of convergence of

$$\lim_{h \rightarrow 0} \frac{e^h + e^{-h} - 2\cosh h}{h} = 0$$

Solution

$$e^h = 1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \dots$$

$$e^{-h} = 1 - h + \frac{h^2}{2!} - \frac{h^3}{3!} + \dots$$

$$\cosh h = 1 + \frac{h^2}{2!} + \frac{h^4}{4!} + \dots$$