Put at the end of this pro

Notice
$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

The Mean Value Theorem implies that if f' exists
$$f[x_0, x_1] = f'(\xi)$$
for some ξ between x_0 and x_1. The following theorem generalizes that

Theorem 3.6 Suppose f has n continuous derivatives and $x_0, \dots, x_n$ are distinct numbers in $[a, b]$. Then $\xi \in (a, b)$ exists with
$$f[x_0, \dots, x_n] = \frac{f^{(n)}(\xi)}{n!}$$

If the interpolating nodes are reordered as $x_n, x_{n-1}, \dots, x_0$ a formula similar to $(\star)$ can be established

$$P_n(x) = f[x_n] + f[x_n, x_{n-1}](x - x_n) + \dots + f[x_n, \dots, x_0](x - x_n) \dots (x - x_1)$$

Def: This form is called the <u>Newton's backward divided-difference formula</u>.