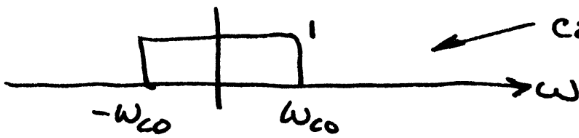
PROBLEM 12.1:

$$h_{bp}(t) = \frac{\sin(\omega_{co}t)}{\pi t}\cos(\omega_0 t)$$

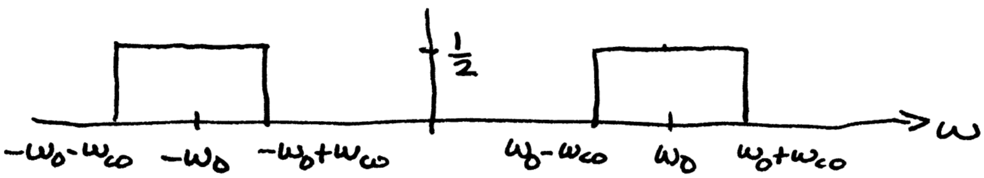
causes frequency shifting

$$u(\omega+\omega_{co}) - u(\omega-\omega_{co})$$



call this $h_{lp}(t)$

$$H_{bp}(j\omega) = \frac{1}{2}H_{lp}(j(\omega+\omega_0)) + \frac{1}{2}H_{lp}(j(\omega-\omega_0))$$



The cutoff frequencies are:

$$\omega_{co1} = \omega_0 - \omega_{co} \qquad \omega_{co2} = \omega_0 + \omega_{co}$$

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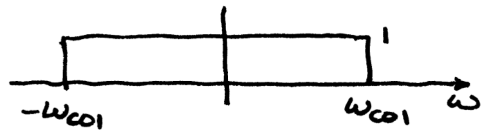
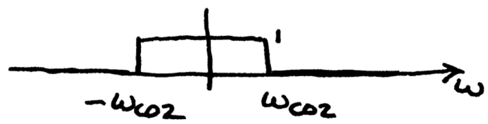
PROBLEM 12.2:

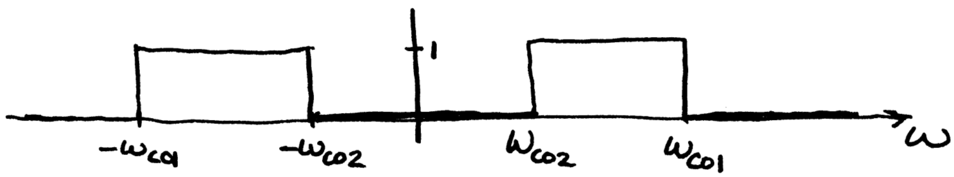
$$\frac{\sin(\omega_{co1} t)}{\pi t} \rightarrow \text{Rectangle}$$



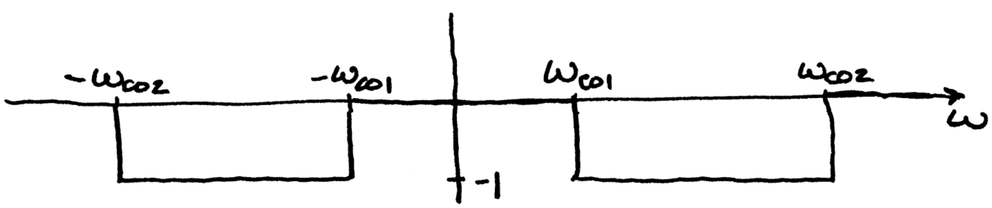
$$\frac{\sin(\omega_{co2} t)}{\pi t} \rightarrow \text{Rectangle}$$



Subtracting the two rectangles gives a BPF



If $\omega_{co2} > \omega_{co1}$ then the BPF has an "amplitude" of -1.



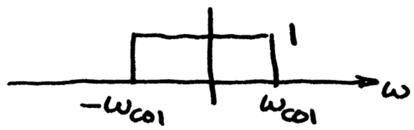
PROBLEM 12.3:

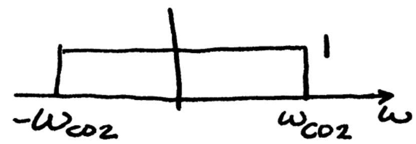
$$h_{bp}(t) = \left[\delta(t) - \frac{\sin(\omega_{co1} t)}{\pi t}\right] * \frac{\sin(\omega_{co2} t)}{\pi t}$$

F.T. 1

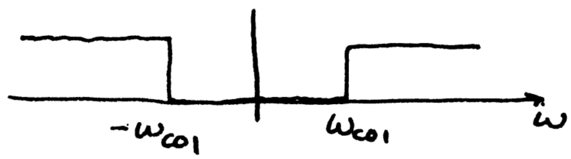
Rectangle

FT Rectangle.





1 − Rectangle



Convolution becomes multiplication in freq. domain

Multiply $(1 - \text{Rect}_1) \cdot \text{Rect}_2$

Only get 1 for $\omega > \omega_{co1}$ and $\omega < \omega_{co2}$

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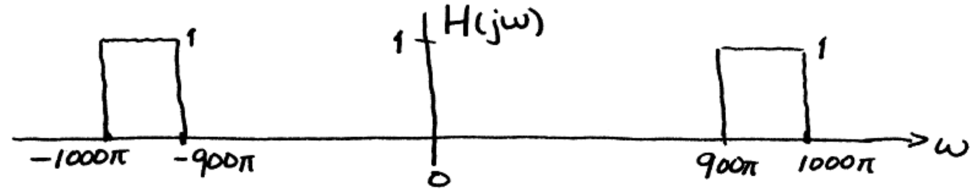
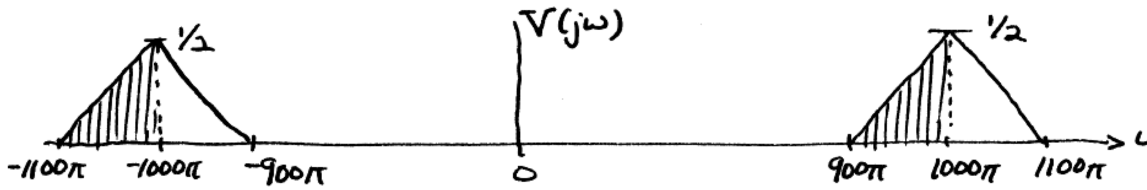
PROBLEM 12.4:

(a)

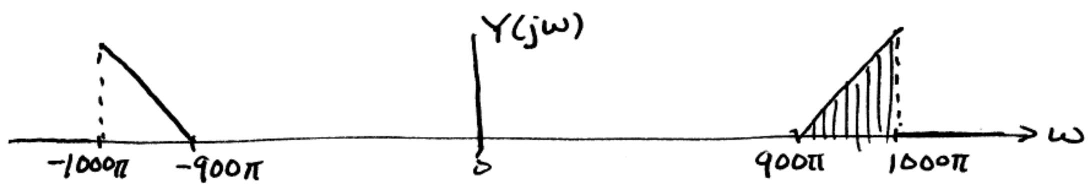


(b) Multiplying by a cosine will cause "frequency shifting"

$$V(j\omega) = \frac{1}{2}X(j(\omega+1000\pi)) + \frac{1}{2}X(j(\omega-1000\pi))$$



(c) Multiply the two graphs



(d) The term "single sideband" (SSB) comes from the following observations:

1. The original $X(j\omega)$ has a positive frequency portion and a negative frequency portion (shaded)
2. In $V(j\omega)$ the shaded portion is called the lower sideband because it is below the "carrier frequency" of 1000π rad/s in this case.
3. The final spectrum (i.e., F.T.) for $Y(j\omega)$ has only one of the sidebands for $w>0$ and the other one for $w<0$. Thus a single sideband remains

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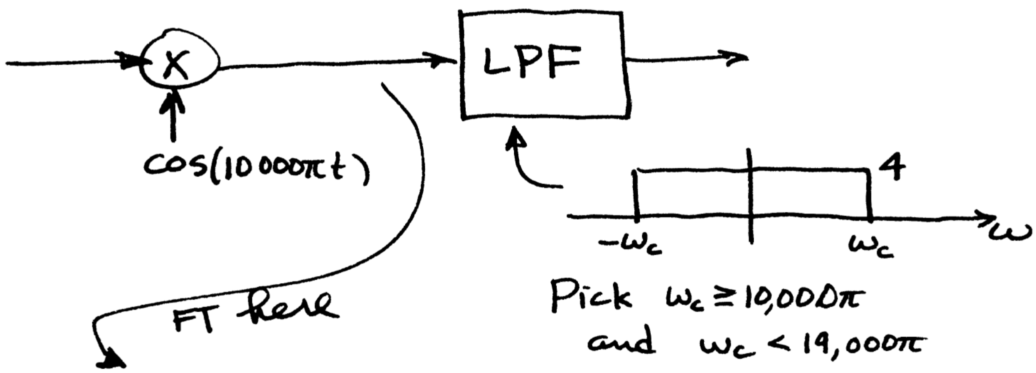
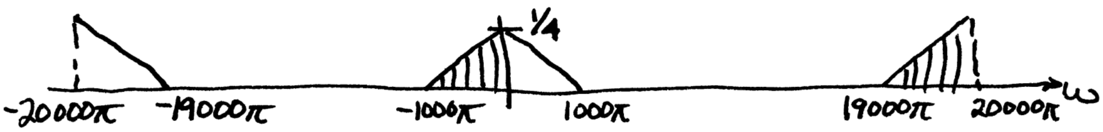
PROBLEM 12.5:

SSB demodulator:

Input spectrum is

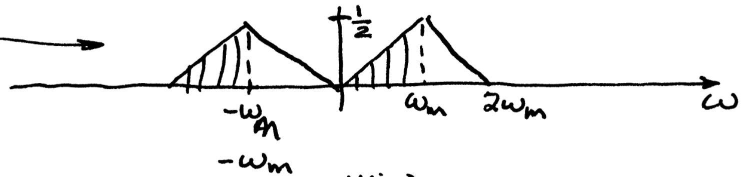


Pick $\omega_c \geq 10{,}000\pi$
and $\omega_c < 19{,}000\pi$

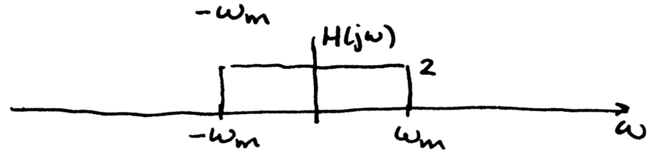


PROBLEM 12.6:

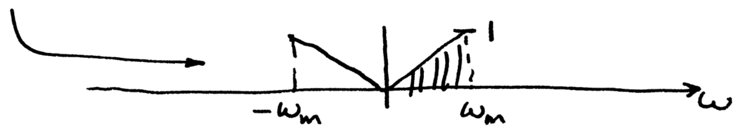
(a) $V(j\omega) = \frac{1}{2}X(j(\omega-\omega_m)) + \frac{1}{2}X(j(\omega+\omega_m))$



(b)

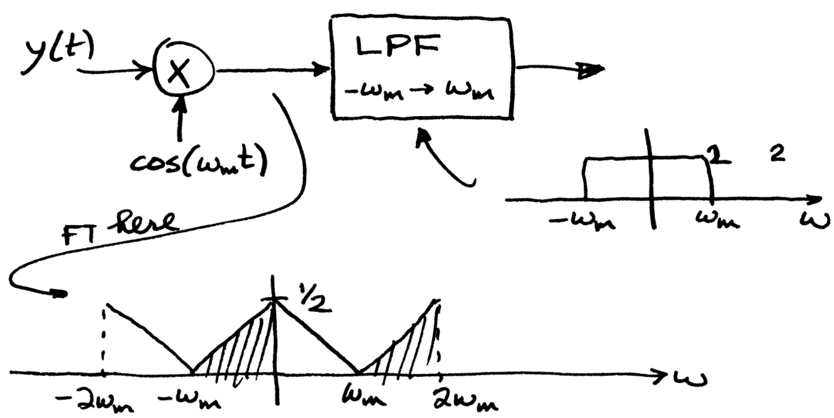


$Y(j\omega) = H(j\omega)V(j\omega)$



(c) The negative-frequency components are moved into the positive frequency region, and vice versa. Also, the high-frequency components are moved to the low frequency region (near $\omega=0$). Likewise, low frequency components are moved to the high frequency region.

(d) Demodulator would be



To recover $x(t)$ we need an ideal LPF that will extract the spectrum from $-\omega_m$ to ω_m.

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PROBLEM 12.7:

a)

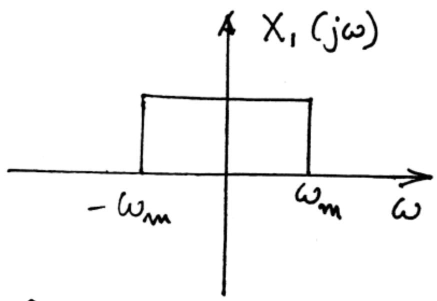
$$x_1(t)\cos(\omega_c t) \longleftrightarrow \frac{1}{2}X_1[j(\omega-\omega_c)] + \frac{1}{2}X_1[j(\omega+\omega_c)]$$

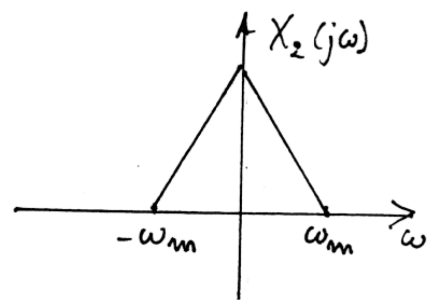
$$x_2(t)\sin(\omega_c t) \longleftrightarrow \frac{1}{2j}X_2[j(\omega-\omega_c)] - \frac{1}{2j}X_2[j(\omega+\omega_c)]$$

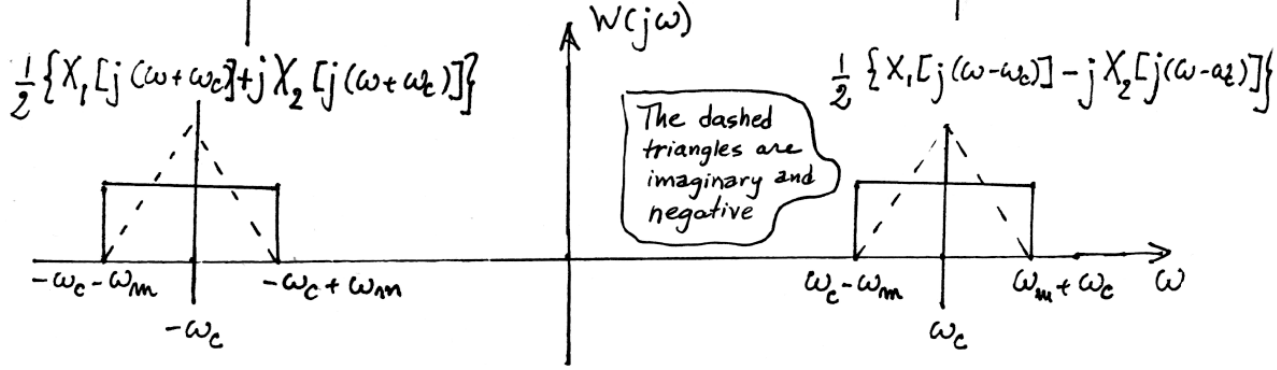
$$w(t) = x_1(t)\cos(\omega_c t) + x_2(t)\sin(\omega_c t) \longleftrightarrow$$

$$W(j\omega) = \frac{1}{2}\left\{X_1[j(\omega-\omega_c)] - jX_2[j(\omega-\omega_c)]\right\} +$$

$$+\frac{1}{2}\left\{X_1[j(\omega+\omega_c)] + jX_2[j(\omega+\omega_c)]\right\}$$







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PROBLEM 12.7 (more):

b) $\omega_a = \omega_c - \omega_m$

$\omega_b = \omega_c + \omega_m$

c) $v(t) = w(t)\sin(\omega_c t) = x_1(t)\sin(\omega_c t)\cos(\omega_c t) +$

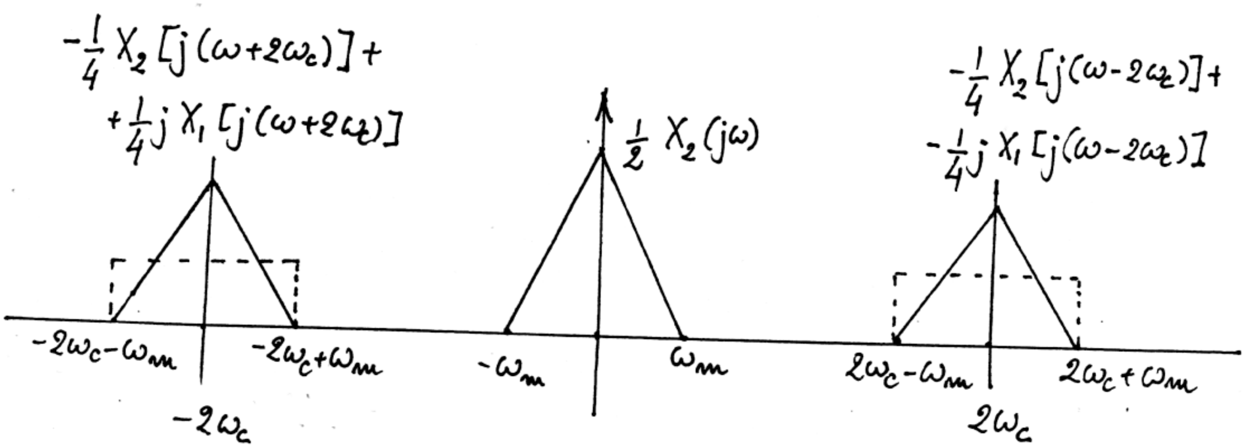
$+ x_2(t)\sin^2(\omega_c t) = \frac{1}{2}x_1(t)\sin(2\omega_c t) +$

$+ \frac{1}{2}x_2(t)[1 - \cos(2\omega_c t)] = \frac{1}{2}x_2(t) +$

$+ \frac{1}{2}x_1(t)\sin(2\omega_c t) - \frac{1}{2}x_2(t)\cos(2\omega_c t)$

$$V(j\omega) = \frac{1}{2}X_2[j\omega] + \frac{1}{4j}\{X_1[j(\omega - 2\omega_c)] +$$

$$- X_1[j(\omega + 2\omega_c)]\} - \frac{1}{4}\{X_2[j(\omega - 2\omega_c)] + X_2[j(\omega + 2\omega_c)]\}$$



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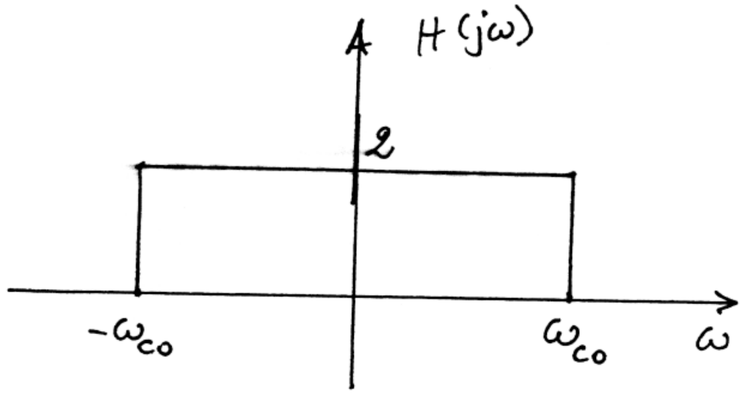
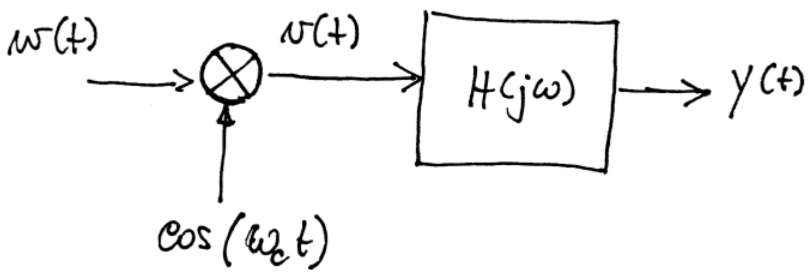
PROBLEM 12.7 (more):

d)



Ideal low-pass filter with gain equal to 2 and $\omega_m \leq \omega_{co} \leq 2\omega_c - \omega_m$ (in fact, a non-ideal low-pass filter will also work).

e)



$$v(t) = x_1(t)\cos^2(\omega_c t) + x_2(t)\sin(\omega_c t)\cos(\omega_c t) =$$

$$= \frac{1}{2}x_1(t)\left[1 + \cos(2\omega_c t)\right] + \frac{1}{2}x_2(t)\sin(2\omega_c t) =$$

$$= \frac{1}{2}x_1(t) + \frac{1}{2}x_1(t)\cos(2\omega_c t) + \frac{1}{2}x_2(t)\sin(2\omega_c t)$$

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PROBLEM 12.8:

a) $\cos\alpha\cos\beta = \dfrac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{2}$

$$\cos(\omega_c t)\cos(\omega_c t+\phi) = \frac{1}{2}\cos(2\omega_c t+\phi) + \frac{1}{2}\cos\phi$$

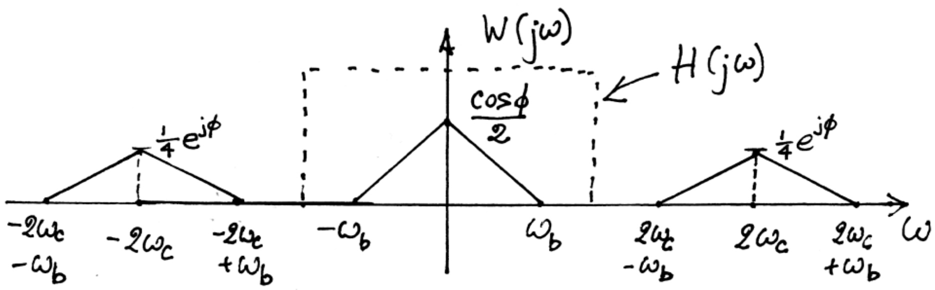
$$w(t) = x(t)\cos(\omega_c t+\phi)\cos(\omega_c t) =$$

$$= \frac{1}{2}x(t)\cos(2\omega_c t+\phi) + \frac{1}{2}x(t)\cos\phi =$$

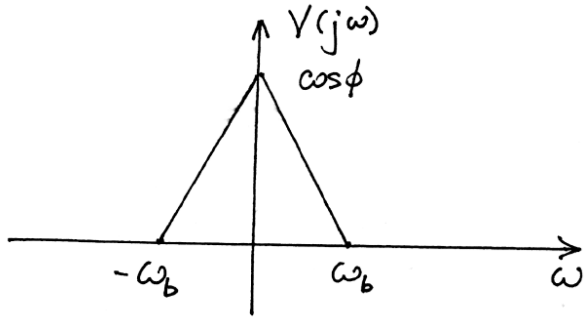
$$= \frac{1}{4}x(t)e^{j\phi}e^{j2\omega_c t} + \frac{1}{4}x(t)e^{-j\phi}e^{-j2\omega_c t} + \frac{1}{2}x(t)\cos\phi$$

b)

$$W(j\omega) = \frac{1}{4}e^{j\phi}X[j(\omega-2\omega_c)] + \frac{1}{4}e^{-j\phi}X[j(\omega+2\omega_c)] + \frac{\cos\phi}{2}X(j\omega)$$



c)



d) $v(t) = x(t)\cos\phi$

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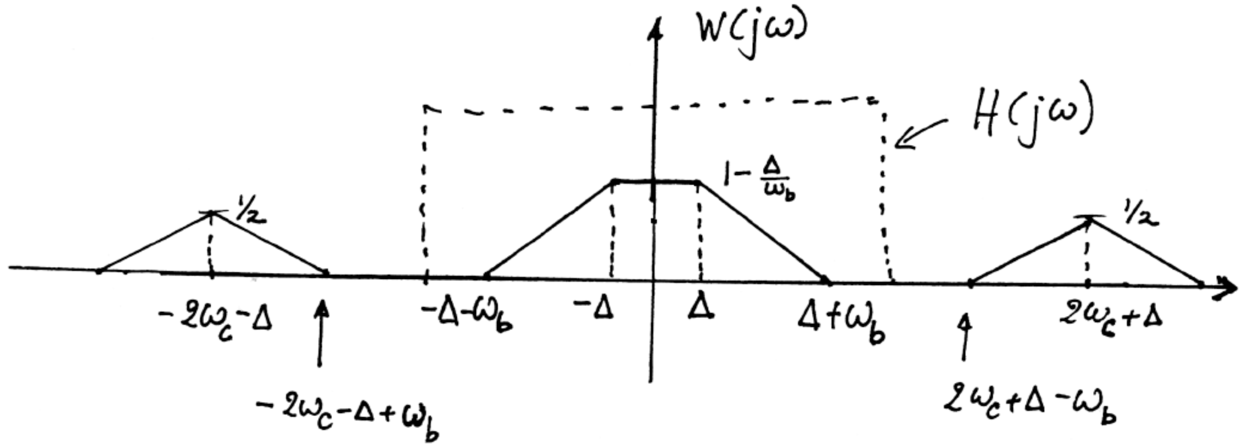
PROBLEM 12.9:

a)

$$w(t) = x(t)\cos(\omega_c t)\cos[(\omega_c+\Delta)t] =$$

$$= \frac{x(t)}{2}\cos(\Delta\cdot t) + \frac{x(t)}{2}\cos[(2\omega_c+\Delta)t]$$



b) $v(t) = x(t)\cos(\Delta\cdot t)$

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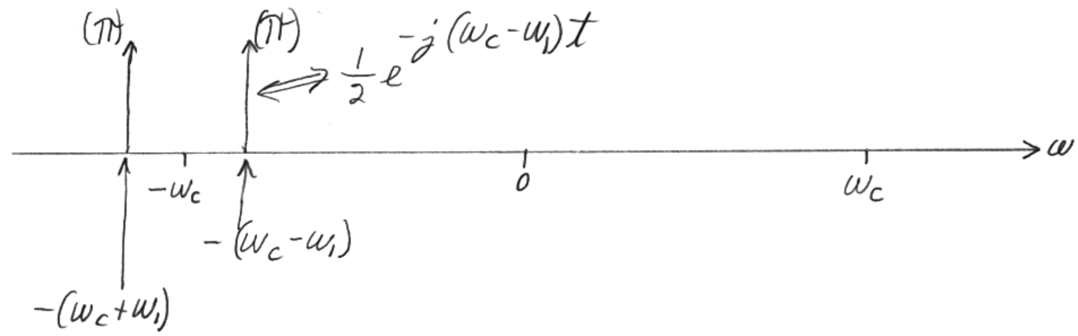
PROBLEM 12.10:

(a)

$$w(t) = \cos(\omega_1 t)\, e^{-j\omega_c t} \Leftrightarrow \frac{1}{2\pi}\left(\pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)\right) * 2\pi\delta(\omega+\omega_c)$$

$$\pi\delta[\omega+(\omega_c+\omega_1)] \;+\; \pi\delta[\omega+(\omega_c-\omega_1)]$$



(b)

$$\omega_0 = \omega_c - \omega_1 \qquad y(t) = \frac{1}{2} e^{-j(\omega_c-\omega_1)t}$$

$$\omega_s \le \omega_c + \omega_1 \qquad \omega_p \ge \omega_c - \omega_1 \ge \omega_0$$

(c)

$$y(t) = \frac{1}{2} e^{-j\omega_0 t}$$

$$s(t) = \frac{1}{2} e^{-j\omega_0 t}\left[\frac{1}{2} e^{+j\omega_0 (t-t_d)}\right] = \frac{1}{4} e^{-j\omega_0 t_d}$$

$$\mathrm{Im}\{s(t)\} = \mathrm{Im}\left\{\frac{1}{4} e^{-j\omega_0 t_d}\right\} = \frac{-1}{4}\sin(\omega_0 t_d)$$

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PROBLEM 12.10 (more):

(d) $x_2(t) = \cos \omega_2 t \qquad \omega_2 = \omega_c + \omega_0$

$$W(t) = \left[\pi\delta(\omega-\omega_2) + \pi\delta(\omega+\omega_2)\right] * \frac{2\pi}{2\pi}\delta(\omega+\omega_c)$$

$$W(j\omega) = \pi\delta[\omega+(\omega_c+\omega_2)] + \pi\delta[\omega+(\omega_c-\omega_2)]$$

$$Y(j\omega) = \pi\delta[\omega+(\omega_c-\omega_2)] = \pi\delta[\omega-\omega_0] \Leftrightarrow e^{j(\omega_0)t}$$

$$s(t) = \frac{1}{2}e^{j\omega_0 t}\left[\frac{1}{2}e^{-j\omega_0(t-t_d)}\right] = \frac{1}{4}e^{j\omega_0 t_d}$$

$$\mathcal{I}m\{s(t)\} = \frac{1}{4}\sin(\omega_0 t_d)$$

(e) if $d(t) > 0$ then ω_2
if $d(t) < 0$ then ω_1

(f) $f_{max} = \omega_c + \omega_0 \quad \therefore f_s = 2\dfrac{(\omega_c+\omega_0)}{2\pi} = \dfrac{\omega_c+\omega_0}{\pi}$

or $f_s = 2(f_c + f_0)$

$f_c = \dfrac{f_s}{2} - f_0$ The output is $\left|\frac{1}{4}\sin(\omega_0 t_d)\right|$.

PROBLEM 12.11

Solution is Under Construction !!!!!!

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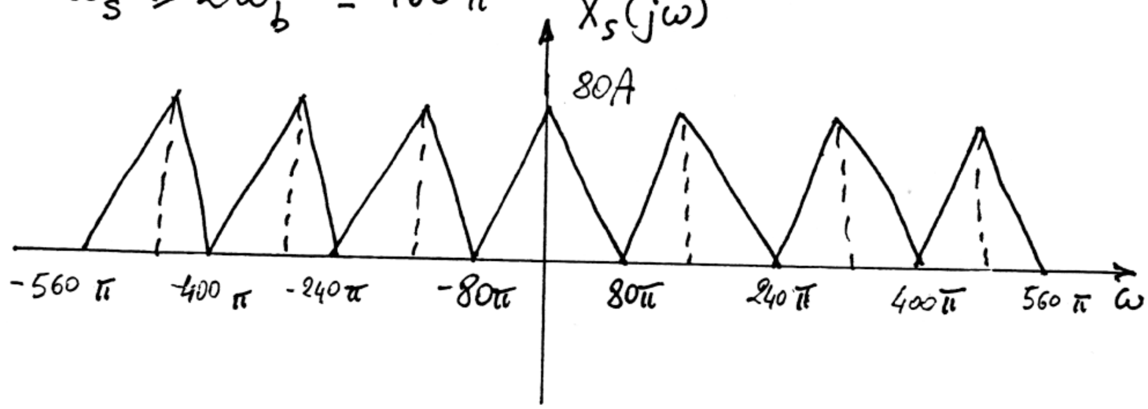
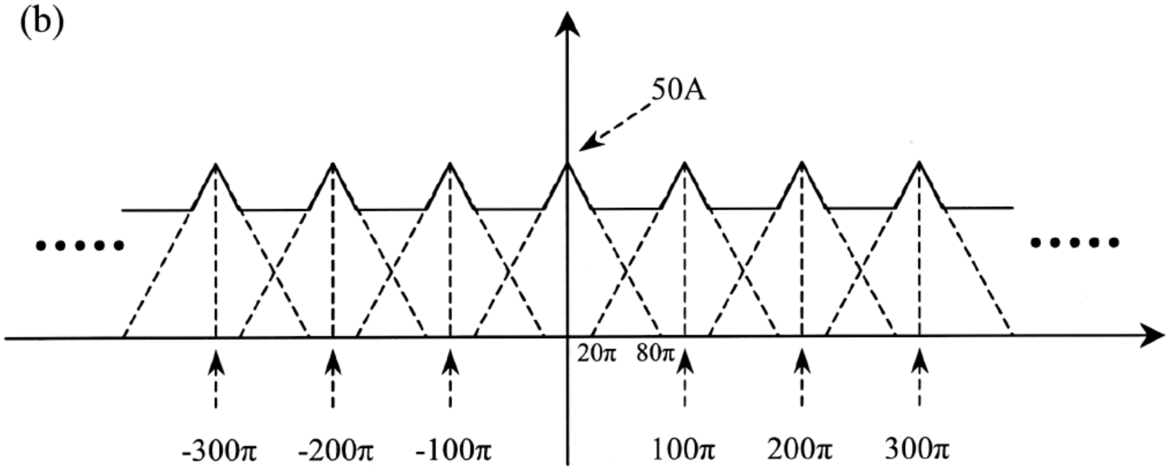
PROBLEM 12.12:

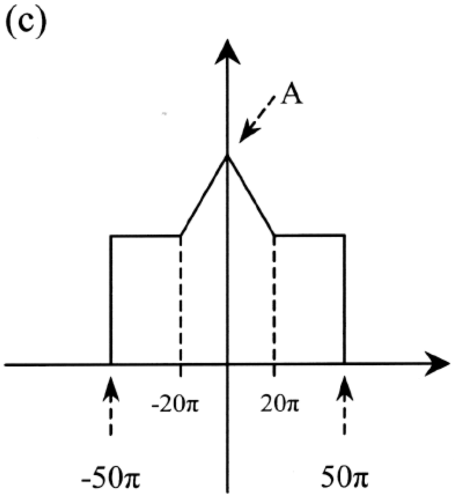
a) $\omega_s \geq 2\omega_b = 160\pi$



(b)



(c)



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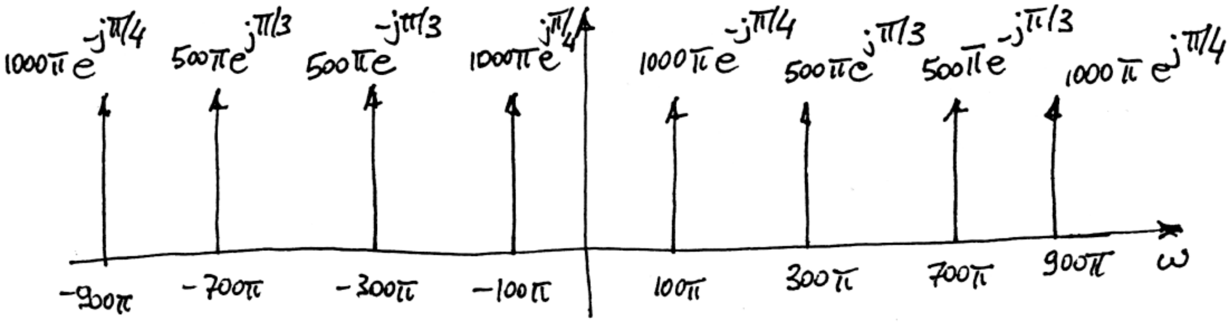
PROBLEM 12.13:

a) $$X(j\omega) = 2\pi\left[e^{-j\pi/4}\delta(\omega-100\pi) + e^{j\pi/4}\delta(\omega+100\pi)\right] +$$
$$+\ \pi\left[e^{j\pi/3}\delta(\omega-300\pi) + e^{-j\pi/3}\delta(\omega+300\pi)\right]$$



$$x_r(t) = x(t)$$

b) $$x_r(t) = 2\cos(100\pi t - \pi/4) + \cos(200\pi t - \pi/3)$$

c) Choose ω_s so that 300π gets aliased to 0,

i.e. $\omega_s = 300\pi = \dfrac{2\pi}{T_s} \Rightarrow T_s = \dfrac{1}{150}$ sec.

$$A = \frac{1}{2}e^{j\pi/3} + \frac{1}{2}e^{-j\pi/3} = \cos(\pi/3) = 0.5\ .$$

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PROBLEM 12.14:

a)
$$\frac{2\pi}{T_s} = \omega_s = 2\,\omega_b = 2000\pi \ \text{rad/sec.}$$

b)
$$\mathcal{H}(\hat{\omega}) = e^{-j10\hat{\omega}}$$

$$\hat{\omega} = \omega T_s$$

$$H_{eff}(j\omega) = e^{-j10\omega T_s} = e^{-j0.01\omega}$$

$$Y_c(j\omega) = H_{eff}(j\omega)\,X_c(j\omega) = e^{-j0.01\omega}\,X_c(j\omega)$$

$$y_c(t) = x_c(t-0.01)$$

c)
$$\mathcal{H}(\hat{\omega}) = \frac{1}{3}\left(1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}\right) = \frac{1}{3}e^{-j\hat{\omega}}\,\frac{\sin\frac{3}{2}\hat{\omega}}{\sin\frac{\hat{\omega}}{2}}$$

$$\hat{\omega} = \omega T_s$$

$$H_{eff}(j\omega) = \frac{1}{3}e^{-j0.001\omega}\,\frac{\sin(0.0015\omega)}{\sin(0.0005\omega)}$$

$$(|\omega| \le 1000\pi)$$

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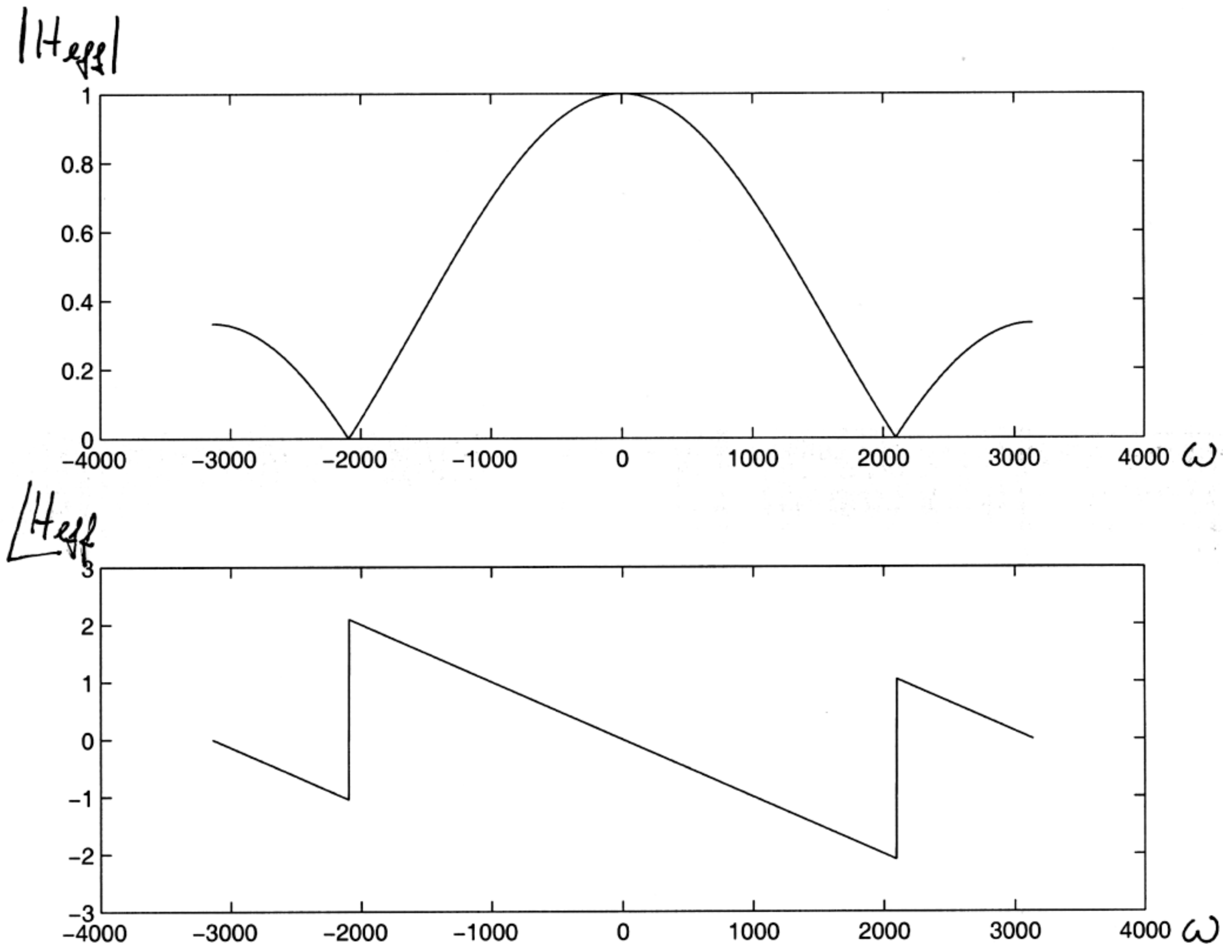
PROBLEM 12.14 (more):



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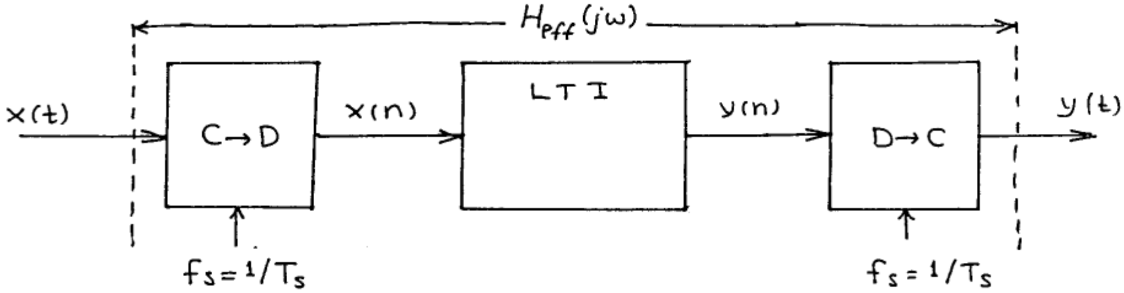
PROBLEM 12.15:



(a) $y(n) = 0.8y(n-1) + x(n) + x(n-2)$

$f_s = 200\text{ Hz}$

$Y(z) = 0.8z^{-1}Y(z) + X(z) + z^{-2}X(z) \Rightarrow$

$$H(z) = \frac{1+z^{-2}}{1-0.8z^{-1}}$$

$$H(e^{j\hat{\omega}}) = \frac{1+e^{-j2\hat{\omega}}}{1-0.8e^{-j\hat{\omega}}} \qquad -\pi < \hat{\omega} < \pi$$

$$H_{eff}(j\omega) = \frac{1+e^{-j2(\omega/200)}}{1-0.8e^{-j(\omega/200)}} \qquad -\pi\cdot 200 < \omega < \pi 200$$

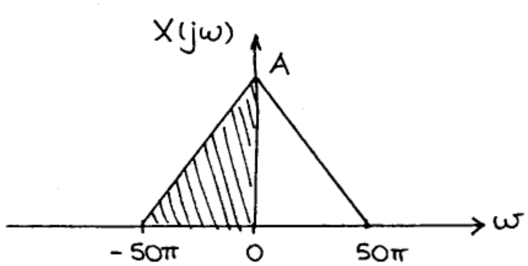
$$\hat{\omega} = 2\pi\hat{f} = 2\pi\frac{f}{f_s} = \omega T_s = \omega/200$$

$$y(t) = 2\,|H_{eff}(j(\omega=100\pi))|\cos(100\pi t + \angle H_{eff}(\omega=100\pi))$$

$$H_{eff}(\omega=100\pi) = \frac{1+e^{-j2\frac{100\pi}{200}}}{1-0.8e^{-j\frac{100\pi}{200}}} = \frac{1+e^{-j\pi}}{1-0.8e^{-j\pi/2}} = 0$$

Therefore $y(t) = 0$

(b)



$f_s \geq 2f_{max}$

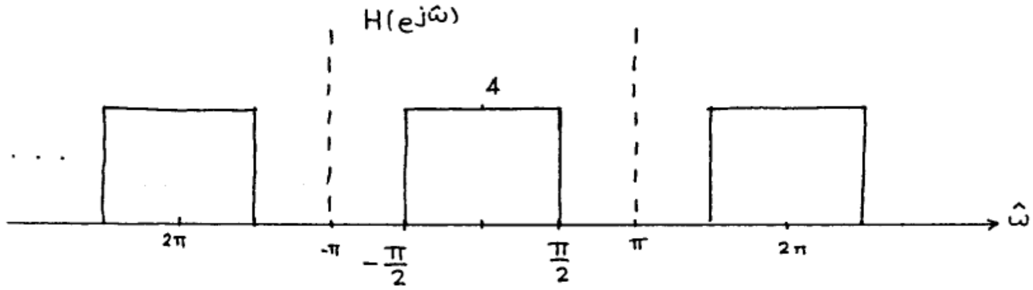
$\omega_{max} = 50\pi = 2\pi(25) \sim$

$f_{max} = 25\text{ Hz}$

$f_s \geq 2\cdot 25 = 50\text{Hz} \sim$

$f_{s\,min} = 50\text{Hz}$

(c)

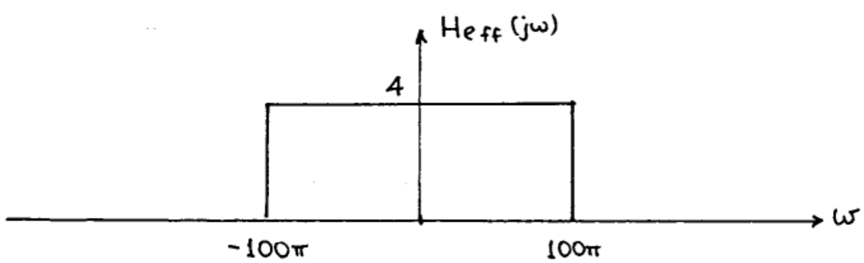


$f_s = 200\,\text{Hz}$

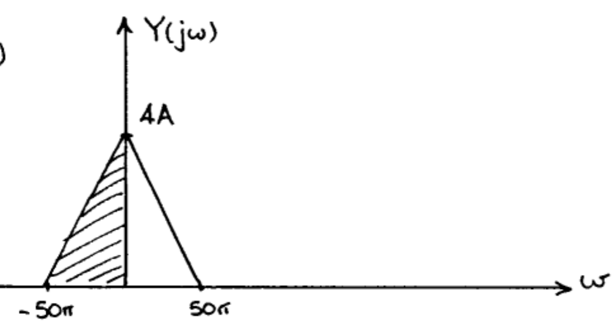
$$H_{eff}(j\omega) = H(e^{j\hat{\omega}})\Big|_{\hat{\omega}=\omega T_s} = H(e^{j\omega/200}) \qquad -\frac{\pi}{2} < \omega T_s = \hat{\omega} < \frac{\pi}{2}$$

$$\rightsquigarrow\ H_{eff}(j\omega) = \begin{cases} 4 & \left|\frac{\omega}{200}\right| < \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases} \ \rightsquigarrow$$

$$H_{eff}(j\omega) = \begin{cases} 4 & |\omega| < 100\pi \\ 0 & |\omega| > 100\pi \end{cases}$$



$$Y(j\omega) = H_{eff}(j\omega)\,X(j\omega)$$



(d) $\quad -\frac{\pi}{2} < \hat{\omega} < \frac{\pi}{2} \ \rightsquigarrow\ -\frac{\pi}{2} < \omega T_s < \frac{\pi}{2} \ \rightsquigarrow\ -\frac{\pi}{2} f_s < \omega < \frac{\pi}{2} f_s$

For $X(j\omega) = Y(j\omega) \rightsquigarrow \quad 50\pi \leqslant \frac{\pi}{2} f_s \Rightarrow f_{s\,min} = \frac{100\pi}{\pi} = 100\,\text{Hz}$