3.2 Divided Differences

a) A practical difficulty with Lagrange interpolation is that since the error term is difficult to apply, the degree of the polynomial needed for the desired accuracy is generally not know until after the computation.

b) The work done in calculating n^{th} degree polynomial does not lessen the work for the computation of the $(n+1)^{st}$ degree polynomial.

1) Divided differences.

Let $P_n(x)$ be the n^{th} Lagrange polynomial such that

$$P_n(x_k) = f_k \qquad k = 0, \ldots, n$$

We want to rewrite $P_n(x)$ in the form

$$P_n(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots + a_n(x-x_0)(x-x_1)\cdots(x-x_{n-1})$$

for appropriate $a_0, a_1, \ldots, a_n$ - constants