PROBLEM 5.1:

If $h[n] = \delta[n-1] - 2\delta[n-4]$

then the filter coefficients are

$$b_k = \{0, 1, 0, 0, -2\}$$

The difference equation is

$$y[n] = x[n-1] - 2x[n-4]$$

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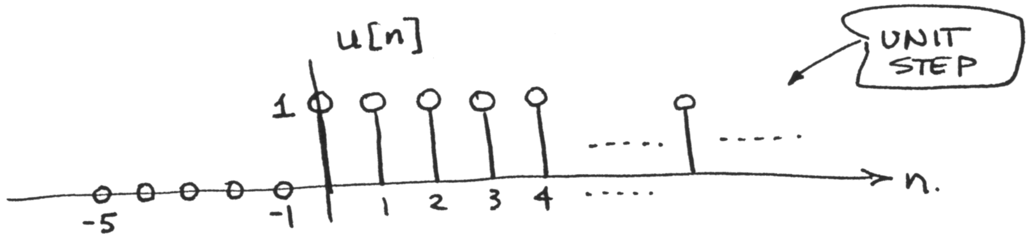
PROBLEM 5.2:

(a)



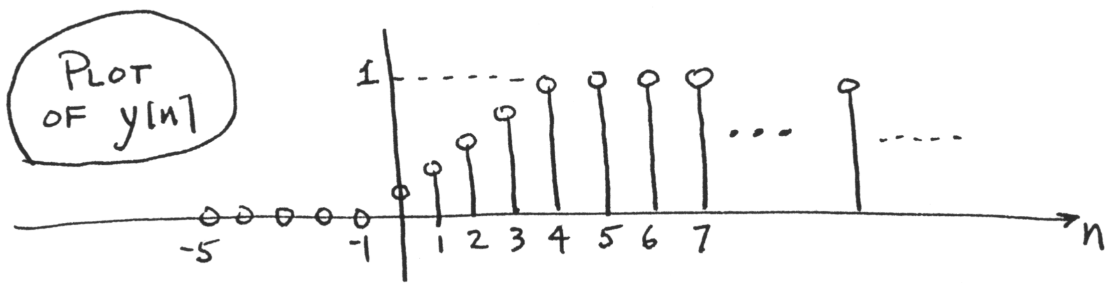
(b) $L=5 \Rightarrow$ avg. 5 points

Make table:

n	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
$x[n]=u[n]$	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
$y[n]$	0	0	0	0	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1	1	1	1	1	1	1

AVG OVER 5-POINTS

(c)



(d) General formula:

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} u[n-k]$$

$$= \frac{1}{L}u[n] + \frac{1}{L}u[n-1] + \frac{1}{L}u[n-2] + \dots + \frac{1}{L}u[n-L+1]$$

For $n<0$, $y[n]=0$

For $n \geq L-1$, $y[n] = \frac{1}{L}(\underbrace{1+1+\dots+1}_{L \text{ times}}) = \frac{1}{L}(L) = 1$

Between, for $0 \leq n \leq L-1$, the output is linearly increasing $y[n] = (n+1)/L$ for $0 \leq n \leq L-1$.

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PROBLEM 5.3:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) MAKE A TABLE:

n	<0	0	1	2	3	4	5	6	7	≥ 8
$x[n]$	0	1	2	3	2	1	1	1	1	1
$y[n]$	0	2	1	2	-1	2	3	1	1	1 ......

$$y[0] = 2x[0] - 3x[-1] + 2x[-2] = 2(1) = 2$$

$$y[1] = 2x[1] - 3x[0] + 2x[-1] = 2(2) - 3(1) = 1$$

$$y[2] = 2x[2] - 3x[1] + 2x[0] = 2(3) - 3(2) + 2(1) = 2$$

$$y[3] = 2(2) - 3(3) + 2(2) = -1$$

$$y[4] = 2(1) - 3(2) + 2(3) = 2$$

$$y[5] = 2(1) - 3(1) + 2(2) = 3$$

$$y[6] = 2(1) - 3(1) + 2(1) = 1$$

$$y[7] = 2(1) - 3(1) + 2(1) = 1$$

$$y[8] = 2(1) - 3(1) + 2(1) = 1$$

(c) Impulse Response

$$h[0] = 2(1) - 3(0) + 2(0) = 2$$

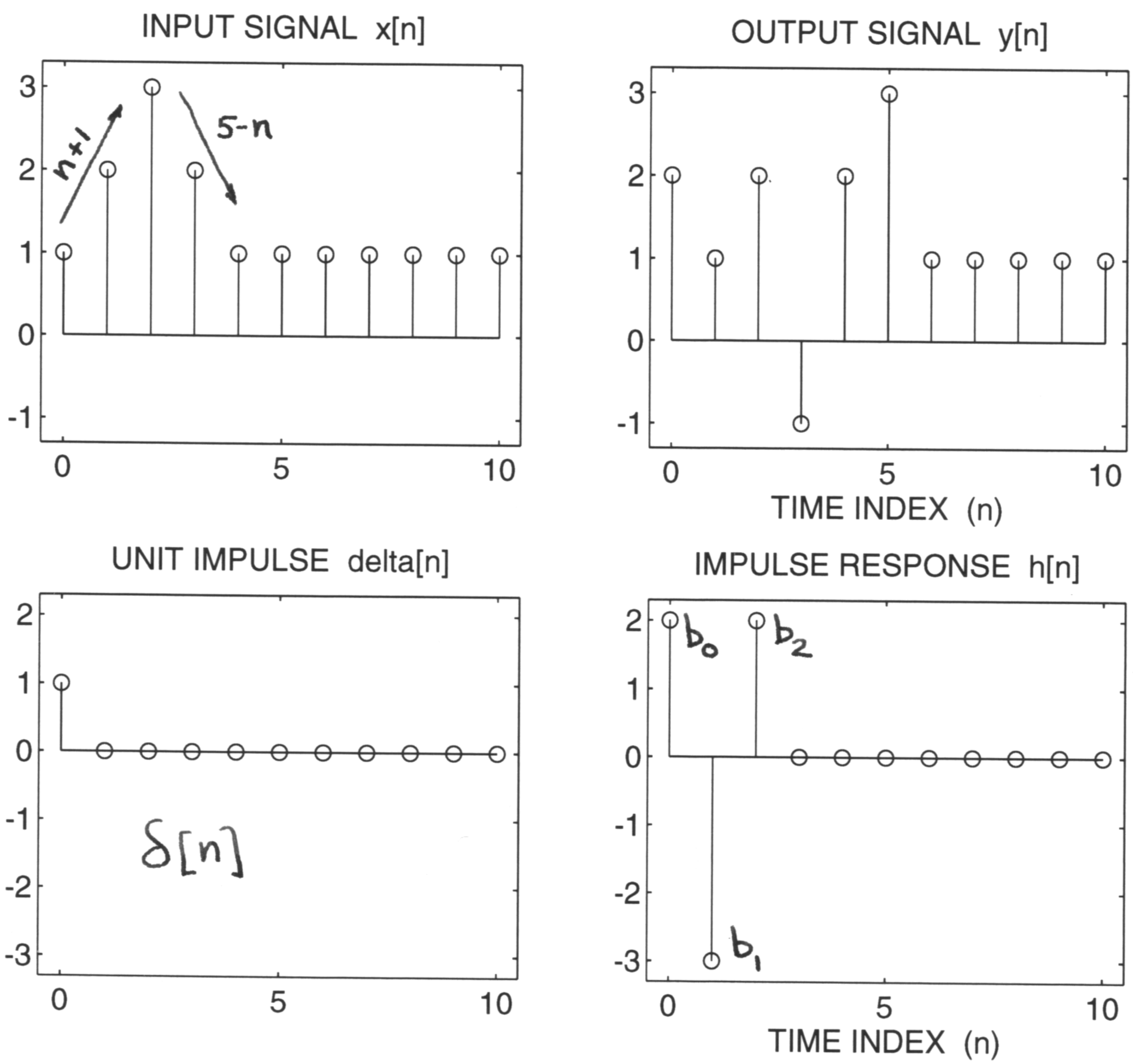
$$h[1] = 2(0) - 3(1) + 2(0) = -3$$

$$h[2] = 2(0) - 3(0) + 2(0) = 2$$

Notice $h[n]$ just "reads out" the filter coefficients:

i.e., $h[n] = b_n$

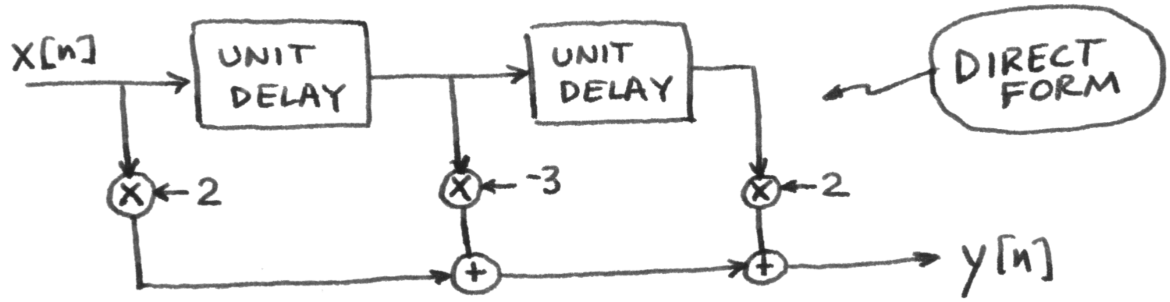
Plots via MATLAB



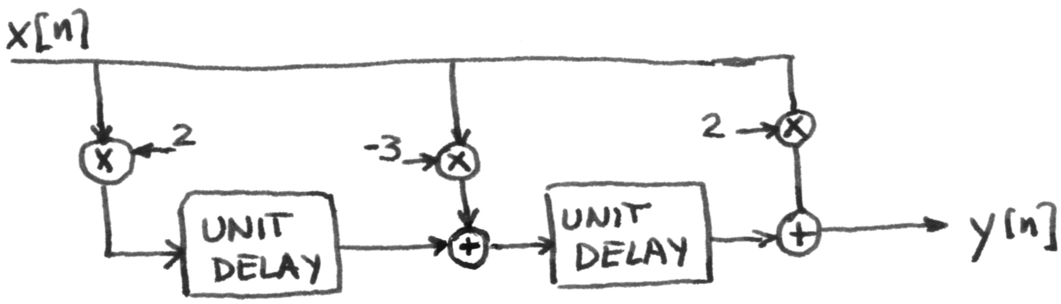
PROBLEM 5.4:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2].$$

(a)



(b) Transposed Form



PROBLEM 5.5:

(a) $y[n] = \sum_{k=0}^{M} b_k x[n-k]$ $\qquad$ $x[n]=0$ for $n<0$ & $n \geq N$

Assume $b_0 \neq 0$ and $b_M \neq 0$

Since $x[n]=0$ for $n<0$, $y[0] = b_0 x[0] + b_1 \cancel{x[-1]}^{\,0} + \text{etc.}$

Thus $y[0] \neq 0$ if $x[0] \neq 0$.

Write out the sum:

$$y[n] = b_M x[n-M] + b_{M-1} x[n-M+1] + b_{M-2} x[n-M+2] + \ldots$$

To find the largest n such that $y[n] \neq 0$, look at the term $x[n-M]$. We need $n-M < N$, otherwise, $x[n-M]=0$

Thus $n < N+M$ is the condition $\qquad P = N+M$

When $n = N+M-1$, then $x[n-M+1] = x[N+M-1-M+1] = x[N] = 0$ so the other terms in the sum drop out.

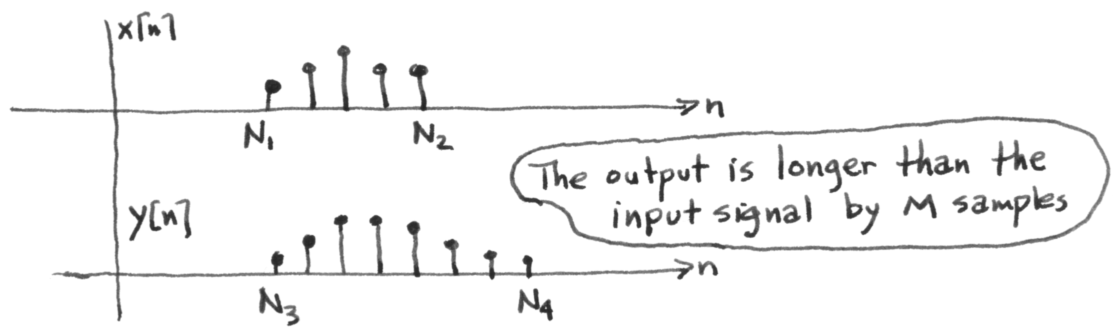
(b) $x[n]=0$ for $n<N_1$ and $n>N_2$

This is just a time shifted version of part (a).

The length of $x[n]$ is $N_2 - N_1 + 1$ which takes the place of N.

The output will start at $n=N_1$ because the time-invariance property applies. The output will end at $n = M + N_2$ because the term $b_M x[n-M]$ will be the last one used in the sum. $\quad N_3 = N_1$ and $N_4 = N_2 + M$

Here's a sketch



PROBLEM 5.6:

Plots for parts (a), (b) and (c) are below.

(d) This general solution will also apply to part (c).

$x[n] = a^n u[n]$

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k} u[n-k]$$

There are 3 cases.

1. $n<0. \Rightarrow y[n]=0$ because $u[n-k]$ is always zero

2. $0 \le n \le L-1$

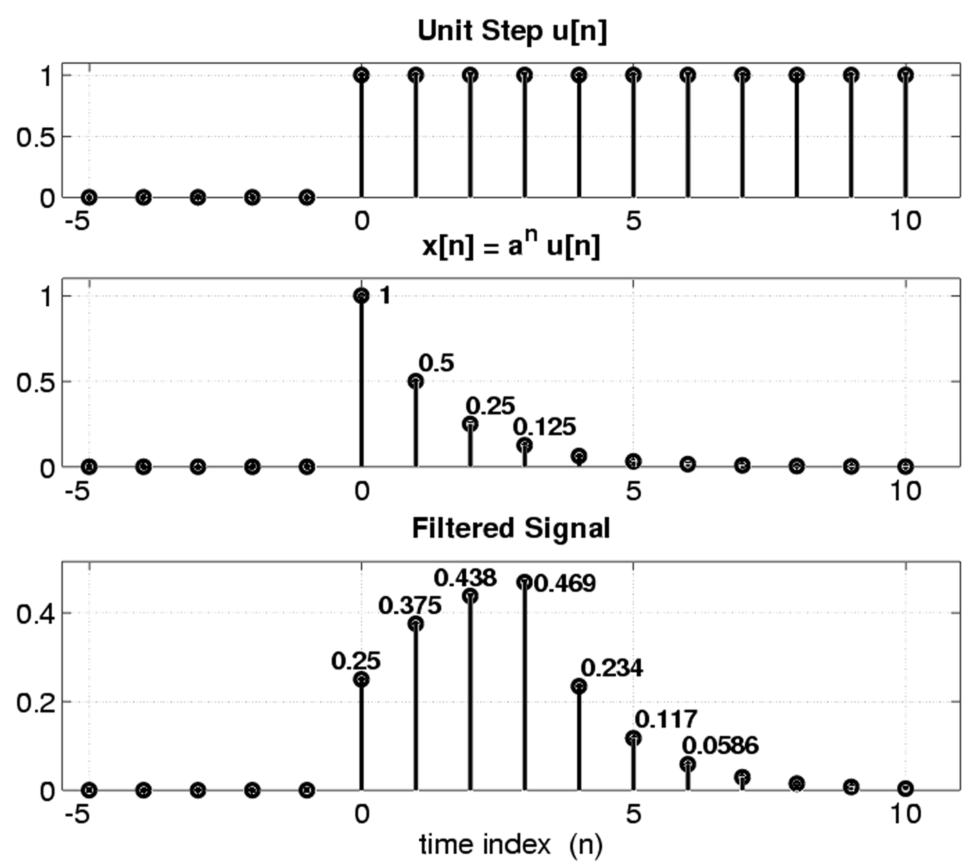
$$y[n] = \frac{1}{L}\sum_{k=0}^{n} a^{n-k} u[n-k] = \frac{a^n}{L}\sum_{k=0}^{n} a^{-k}$$

$$y[n] = \frac{a^n}{L}\left(\frac{1-a^{-n-1}}{1-a^{-1}}\right) = \frac{1}{L}\left(\frac{a^{n+1}-1}{a-1}\right)$$

3. $n \ge L$

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} a^{n-k} \cancel{u[n-k]}^{\,1} = \frac{a^n}{L}\sum_{k=0}^{L-1} a^{-k}$$

$$= \frac{a^n}{L}\left(\frac{1-a^{-L}}{1-a^{-1}}\right) = \frac{a^n}{L}\left(\frac{a^L-1}{a^L-a^{L-1}}\right) \quad \text{for } n \ge L.$$



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PROBLEM 5.7:

$$h[n] = 3\delta[n] + 7\delta[n-1] + 13\delta[n-2] + 9\delta[n-3] + 5\delta[n-4]$$

The impulse response contains the values of the filter coefficients because

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

Thus,

$$b_0 = 3, \quad b_1 = 7, \quad b_2 = 13, \quad b_3 = 9, \quad b_4 = 5$$

PROBLEM 5.8:

Use convolution

n:	...	-2	-1	0	1	2	3	4	....
$x[n]$:		0	1	0	1	0	1	0	....
$h[n]$:				13	-13	13			
	....	0	13	0	13	0	13	0	....
	....	-13	0	-13	0	-13	0	-13	....
	....	0	13	0	13	0	13	0	.....
		-13	26	-13	26	-13	26	-13	
				$\uparrow$ $n=0$					

$$\Rightarrow \quad y[n] = \begin{cases} -13 & \text{for } n \text{ even} \\ 26 & \text{for } n \text{ odd} \end{cases}$$

PROBLEM 5.9:

Linearity?

(a) YES.

Let $x[n] = \alpha_1 x_1[n] + \alpha_2 x_2[n]$ $\qquad x_1[n] \to y_1[n]$, $\; x_2[n] \to y_2[n]$

$$\Rightarrow y[n] = (\alpha_1 x_1[n] + \alpha_2 x_2[n])\cos(0.2\pi n)$$

$$= \alpha_1 \underbrace{x_1[n]\cos(0.2\pi n)}_{y_1[n]} + \alpha_2 \underbrace{x_2[n]\cos(0.2\pi n)}_{y_2[n]}$$

(b) YES.

$$y[n] = (\alpha_1 x_1[n] - \alpha_2 x_2[n]) - (\alpha_1 x_1[n-1] + \alpha_2 x_2[n-1])$$

$$= \alpha_1 (\underbrace{x_1[n] - x_1[n-1]}_{y_1[n]}) + \alpha_2 (\underbrace{x_2[n] - x_2[n-1]}_{y_2[n]})$$

(c) NO.

Let $x_1[n] = \delta[n]$ and $x_2[n] = -2\delta[n]$.

$\hookrightarrow y_1[n] = \delta[n]$ $\qquad \hookrightarrow y_2[n] = |x_2[n]| = 2\delta[n]$

Let $x[n] = x_1[n] + x_2[n] = \delta[n] - 2\delta[n] = -\delta[n]$

$\hookrightarrow y[n] = |x[n]| = \delta[n] \longleftrightarrow y_1[n] + y_2[n] = \delta[n] + 2\delta[n] = 3\delta[n]$

NOT EQUAL!

(d) NO! if $B \neq 0$

if $x_1[n] \to y_1[n]$, test $2x_1[n] \to 2y_1[n]$.

$$A(2x_1[n]) + B = 2(Ax_1[n] + B) - B \neq 2y_1[n]$$

TIME-INVARIANT?

(a) NO!

Let $x[n] = \delta[n]$, then $y[n] = \delta[n]\cos(0.2\pi n) = \delta[n]$

EVAL AT $n=0$

Try $x[n-1] = \delta[n-1]$, then output is

$$\delta[n-1]\cos(0.2\pi n) = \cos(0.2\pi)\,\delta[n-1].$$

BUT $\cos(0.2\pi)\,\delta[n-1] \neq y[n-1] = \delta[n-1]$

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PROBLEM 5.9 (more):

TIME-INVARIANT?

(b) Yes.

If $x[n] \longrightarrow y[n]$, Let $v[n] = x[n-n_0]$

OUTPUT $= v[n] - v[n-1] = x[n-n_0] - x[n-n_0-1]$

This is the same as $y[n-n_0] = x[n-n_0] - x[n-n_0-1]$

(c) YES.

Output depends only on x[ ] at "n", so $y[n-n_0] = |x[n-n_0]|$

(d) Yes

$y[n-n_0] = Ax[n-n_0] + B$ is always true.

CAUSAL?

(a) YES.

y[n] at $n=n_0$ depends only on x[n] at $n=n_0$, and not on past <u>or</u> future values.

(b) Yes.

y[n] at $n=n_0$ depends only on x[n] at $n=n_0$ & $n=n_0-1$ so it only uses the "present" and the "past."

(c) YES

y[n] at $n=n_0$ depends only on x[n] at $n=n_0$.

$$y[n_0] = |x[n_0]|$$

(d) YES

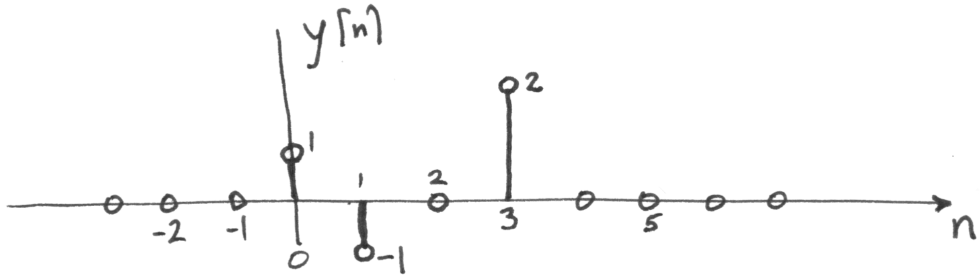
y[n] at $n=n_0$ depends only on x[n] at $n=n_0$.

$$y[n_0] = Ax[n_0] + B$$

PROBLEM 5.10:

$$x[n] = \delta[n] - \delta[n-1] \longrightarrow y[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$$
$$x[n] = \cos(\pi n/2) \longrightarrow y[n] = 2\cos(\pi n/2 - \pi/4)$$

(a) Make a plot of the signal: $y[n] = \delta[n] - \delta[n-1] + 2\delta[n-3]$.



(b) Use linearity and time-invariance to find the output of the system when the input is

$$x[n] = 7\delta[n] - 7\delta[n-2]$$

In order to use Linearity & Time-Inv, we need to express $x[n]$ in terms of known signal.

Let $x_1[n] = \delta[n] - \delta[n-1]$

Then $x[n] = 7\delta[n] - 7\delta[n-2] = 7x_1[n] + 7x_1[n-1]$

Because $x_1[n-1] = \delta[n-1] - \delta[n-2]$.

Now, LTI system $\Rightarrow$

$$7x_1[n] \longrightarrow 7\delta[n] - 7\delta[n-1] + 14\delta[n-3]$$
$$7x_1[n-1] \longrightarrow 7\delta[n-1] - 7\delta[n-2] + 14\delta[n-4]$$

Add them together:

$$x[n] \longrightarrow 7\delta[n] - 7\delta[n-2] + 14\delta[n-3] + 14\delta[n-4]$$

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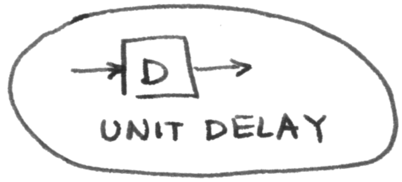
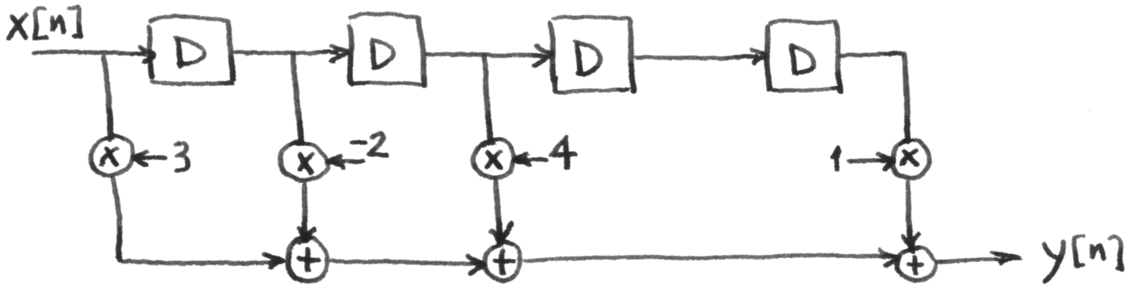
PROBLEM 5.11:

$$h[n] = 3\delta[n] - 2\delta[n-1] + 4\delta[n-2] + \delta[n-4]$$

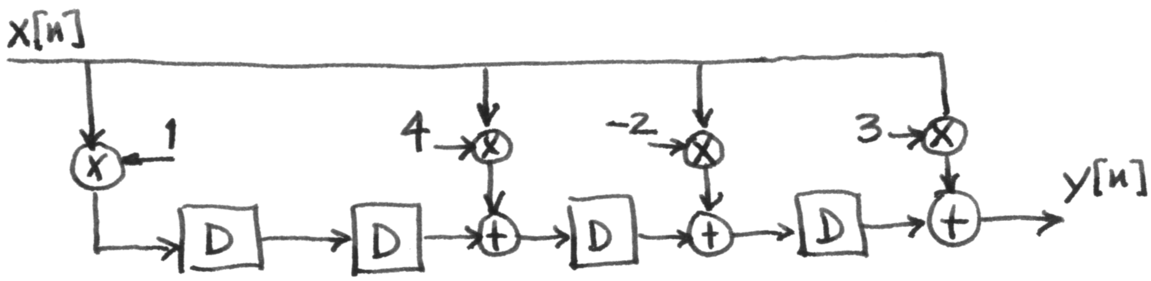
$$\Rightarrow \; y[n] = 3x[n] - 2x[n-1] + 4x[n-2] + x[n-4]$$

(a) Direct Form:





(b) Transposed Direct Form:



PROBLEM 5.12:

$$x_1[n] = u[n] \longrightarrow y_1[n] = \delta[n] + 2\delta[n-1] - \delta[n-2]$$

$$x_2[n] = 3u[n] - 2u[n-4]$$

Use linearity and time-invariance:

$$y_2[n] = 3y_1[n] - 2y_1[n-4]$$

$$= 3\delta[n] + 6\delta[n-1] - 3\delta[n-2] - 2\delta[n-4]$$
$$- 4\delta[n-5] + 2\delta[n-6].$$

List of values:

n	<0	0	1	2	3	4	5	6	≥ 7
$y_2[n]$	0	3	6	-3	0	-2	-4	2	0

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PROBLEM 5.13:

(a) $y[n] = x[n] - ax[n-1]$

$= a^n u[n] - a(a^{n-1} u[n-1])$

$= a^0 \delta[n] + a^n u[n-1] - a^n u[n-1] = \delta[n]$

(b) Express $x[n]$ as a sum: $x[n] = a^n u[n] + (-a^n u[n-10])$

Because the FIR filter is an LTI system, we can find the output for $-a^n u[n-10]$ and add it to the result from part (a)

$y_2[n] = -a^n u[n-10] - a(-a^{n-1} u[n-11])$

$= -a^{10} \delta[n-10] - a^n u[n-11] + a^n u[n-11]$

$= -a^{10} \delta[n-10]$

$\therefore \quad y[n] = \delta[n] - a^{10} \delta[n-10]$

from part (a)

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PROBLEM 5.14:

(a) $h[n] = \delta[n-2] \Rightarrow$ filter is a delay by 2

$y[n] = u[n-3] - u[n-6]$

To find $x[n]$ we need to "un-delay" $y[n]$.

$\Rightarrow x[n] = u[n-1] - u[n-4]$

(b) First-difference FIR $\Rightarrow h[n] = \delta[n] - \delta[n-1]$

The first-difference filter has a nonzero output at n when $x[n]$ & $x[n-1]$ are not equal.

If $y[n] = \delta[n] - \delta[n-4]$, then the input $x[n]$ changes value at $n=0$ and $n=4$. At $n=0$, it jumps up by one; at $n=4$, it jumps down.

$\Rightarrow x[n] = u[n] - u[n-4]$

jump up by one

jump down

(c) 4-pt averager: $y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$

If $y[n] = -5\delta[n] - 5\delta[n-2]$

$y[0] = -5 = \frac{1}{4}\left(x[0] + x[-1] + x[-2] + x[-3]\right)$

** if we assume $x[n]=0$ for $n<0$, then $x[0] = -20$

$y[1] = 0 = \frac{1}{4}\left(x[1] + x[0] + x[-1] + x[-2]\right) = \frac{1}{4}x[1] - 5$

$\Rightarrow x[1] = 20$

$$\left.\begin{aligned} y[2] = -5 &= \tfrac{1}{4}\left(x[2] + x[1] + x[0] + x[-1]\right) \\ &= \tfrac{1}{4}\left(x[2] + 20 - 20 + 0\right) = \tfrac{1}{4}x[2] \end{aligned}\right\} x[2] = -20$$

$y[3] = 0 = \frac{1}{4}\left(x[3] + x[2] + x[1] + x[0]\right) \Rightarrow x[3] = -20$

$$\Rightarrow x[n] = \begin{cases} 0 & \text{for } n < 0 \\ -20 & \text{for } n \text{ even} \\ 20 & \text{for } n \text{ odd} \end{cases}$$

PROBLEM 5.15:

(a) $x[n] = u[n]$ and $y[n] = u[n-1]$

We need a "delay by one".

$\Rightarrow h[n] = \delta[n-1]$

(b) $x[n] = u[n]$ and $y[n] = \delta[n]$

Since $u[n]$ jumps from 0 to 1 at $n=0$, we need a filter that detects jumps. This can be done with a first-difference filter.

$h[n] = \delta[n] - \delta[n-1]$

(c) $x[n] = (\frac{1}{2})^n u[n]$ and $y[n] = \delta[n-1]$

Use the convolution sum to write linear equations:

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k].$$

Note: $x[n]=0$ for $n<0$

$y[0] = h[0]\,x[0] + h[1]\,x[-1] + \ldots$

$0 = h[0](\frac{1}{2})^0 = h[0] \quad \Rightarrow h[0] = 0$

$y[1] = h[0]\,x[1] + h[1]\,x[0] + h[2]\,\cancel{x[-1]}^{\,0} + \ldots$

$1 = 0 + h[1](\frac{1}{2})^0 = h[1] \quad \Rightarrow h[1] = 1$

$y[2] = h[0]\,x[2] + h[1]\,x[1] + h[2]\,x[2]$

$0 = 0 + 1(\frac{1}{2})^1 + h[2](\frac{1}{2})^0$

$0 = \frac{1}{2} + h[2] \quad \Longrightarrow \quad h[2] = -\frac{1}{2}$

$y[3] = h[0]\,x[3] + h[1]\,x[2] + h[2]\,x[1] + h[3]\,x[0].$

$0 = 0 + 1(\frac{1}{2})^2 - \frac{1}{2}(\frac{1}{2})^1 + h[3](\frac{1}{2})^0$

$0 = 0 + \frac{1}{4} - \frac{1}{4} + h[3] \quad \Rightarrow \quad h[3] = 0$

Similarly for $n > 3$

$\therefore\ h[n] = \delta[n-1] - \frac{1}{2}\delta[n-2]$

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PROBLEM 5.16:

Sometimes it is not possible to solve the *deconvolution* process for a given input-output pair. For example, prove that there is no FIR filter that can process the input $x[n] = \delta[n]+\delta[n-1]$ to give the output $y[n] = \delta[n]$.

Solution: The *deconvolution* filter that turns $x[n]$ into $y[n]$ must have an impulse response $h[n]$ satisfying

$$y[n] = h[n] * x[n] \qquad \text{for all } n$$

The method of proof will be to assume that $h[n]$ is the impulse response of an FIR filter and show that we get a contradiction. If $h[n]$ has finite length, then the most general statement we can make about $h[n]$ is that it's zero outside of a finite region, i.e.,

$$h[n] = 0 \qquad \text{for } n < N_1 \text{ or } n > N_2 \tag{1}$$

Note: it is not necessary to assume that $h[n]$ is the impulse response of a causal filter, but if it were then N_1 would be greater than or equal to zero.

We could consider several separate cases depending on whether N_1 is less than zero, equal to zero, or greater than zero. However, the most general case to consider would be the one where $N_1 < 0$, so we now write out a few terms of the convolution equation to see the general form:

$$\begin{aligned}
y[N_1] = 0 = h[N_1] + h[N_1-1] = h[N_1] + 0 \quad &\Longrightarrow \quad h[N_1] = 0 \\
y[N_1+1] = 0 = h[N_1+1] + h[N_1] \quad &\Longrightarrow \quad h[N_1+1] = 0 \\
y[N_1+2] = 0 = h[N_1+2] + h[N_1+1] \quad &\Longrightarrow \quad h[N_1+2] = 0 \\
\vdots \qquad\qquad \vdots \qquad & \\
y[-1] = 0 = h[-1] + h[-2] \quad &\Longrightarrow \quad h[-1] = 0 \\
y[0] = 1 = h[0] + h[-1] = h[0] + 0 \quad &\Longrightarrow \quad h[0] = 1 \\
y[1] = 0 = h[1] + h[0] = h[1] + 1 \quad &\Longrightarrow \quad h[1] = -1 \\
y[2] = 0 = h[2] + h[1] = h[2] - 1 \quad &\Longrightarrow \quad h[2] = 1 \\
\vdots \qquad\qquad \vdots \qquad & \\
y[N_2-1] = 0 = h[N_2-1] + h[N_2-2] \quad &\Longrightarrow \quad h[N_2-1] = -1 \\
y[N_2] = 0 = h[N_2] + h[N_2-1] \quad &\Longrightarrow \quad h[N_2] = 1 \\
y[N_2+1] = 0 = h[N_2+1] + h[N_2] \quad &\Longrightarrow \quad h[N_2+1] = -1 \\
y[N_2+2] = 0 = h[N_2+2] + h[N_2+1] \quad &\Longrightarrow \quad h[N_2+2] = 1 \\
\vdots \qquad\qquad \vdots \qquad &
\end{aligned}$$

where we have assumed that N_2 is an even integer.

The solution for the values of $h[n]$ is done by solving the equations one at a time from top to bottom. The final two equations show that $h[n]$ will be nonzero even when $n > N_2$ and thus provide the *contradiction* of the FIR assumption in equation (1). Hence, we are able to conclude that there is no FIR filter that can process the input $x[n] = \delta[n] + \delta[n-1]$ to give the output $y[n] = \delta[n]$.

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PROBLEM 5.17:

(a)
$$h_1[n] = \delta[n] - \delta[n-1]$$
$$h_2[n] = \delta[n] + \delta[n-2]$$
$$h_3[n] = \delta[n-1] + \delta[n-2]$$

(b) The overall $h[n]$ is the convolution of the $h_i[n]$.

$$h[n] = h_1[n] * h_2[n] * h_3[n]$$

$$h_1[n] * h_2[n] = (\delta[n] - \delta[n-1]) * (\delta[n] + \delta[n-2])$$
$$= \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$$

Now convolve with $h_3[n]$

$$\begin{array}{cccccc}
1 & -1 & 1 & -1 & & \\
0 & 1 & 1 & & & \\
\hline
0 & 0 & 0 & 0 & & \\
 & 1 & -1 & 1 & -1 & \\
 & & 1 & -1 & 1 & -1 \\
\hline
0 & 1 & 0 & 0 & 0 & -1 \\
n=0 & \uparrow_{n=1} & & & & \uparrow_{n=5}
\end{array}$$

$$\boxed{h[n] = \delta[n-1] - \delta[n-5]}$$

(c)
$$y[n] = h[n] * x[n]$$
$$= (\delta[n-1] - \delta[n-5]) * x[n]$$
$$y[n] = x[n-1] - x[n-5]$$

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PROBLEM 5.18:

The MATLAB program has two filters that are added together, and then filtered again

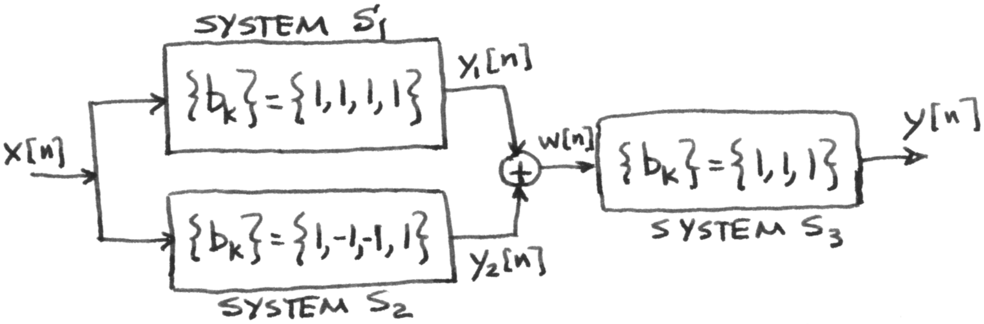
$$y_1[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

$$y_2[n] = x[n] - x[n-1] - x[n-2] + x[n-3]$$

$$w[n] = y_1[n] + y_2[n]$$

$$y[n] = w[n] + w[n-1] + w[n-2]$$

(a)



$$S_1: \; h_1[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

$$S_2: \; h_2[n] = \delta[n] - \delta[n-1] - \delta[n-2] + \delta[n-3]$$

$$S_3: \; h_3[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

(b) When $x[n] = \delta[n]$, $w[n] = h_1[n] + h_2[n]$

$$= 2\delta[n] + 2\delta[n-3]$$

Then $y[n] = h_3[n] * w[n]$

$$= 2\delta[n] + 2\delta[n-1] + 2\delta[n-2] + 2\delta[n-3] + 2\delta[n-4] + 2\delta[n-5]$$

The overall difference equation is obtained by noting that the filter coeffs are equal to the impulse response values: $b_k = h[n]\big|_{n=k}$

$$y[n] = 2x[n] + 2x[n-1] + 2x[n-2] + 2x[n-3] + 2x[n-4] + 2x[n-5]$$