PROBLEM 6.1:

$$x[n] = e^{-j\pi/2} e^{j0.4\pi n}$$

$$e^{-j\pi/2} = -j$$

$$y[n] = x[n] - x[n-1]$$

$$= e^{-j\pi/2} e^{j0.4\pi n} - e^{-j\pi/2} e^{j0.4\pi(n-1)}$$

$$= e^{-j\pi/2} e^{j0.4\pi n} \big(\underbrace{1 - e^{-j0.4\pi}}_{= 1.176 e^{j0.3\pi}} \big)$$

ANGLE = 54° or 0.942 rads

$$= 1.176 e^{-j0.2\pi} e^{j0.4\pi n}$$

$$A = 1.176 \qquad \varphi = -0.2\pi = -0.628 \text{ rads, or } -36^\circ$$

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PROBLEM 6.2:

$$y[n] = (x[n])^2$$

(a) $x[n] = Ae^{j\varphi}e^{j\hat{\omega}n}$

$$y[n] = (Ae^{j\varphi}e^{j\hat{\omega}n})^2 = A^2e^{j2\varphi}e^{j2\hat{\omega}n}$$

(b) NO.

The output cannot be written as

$$y[n] = \mathcal{H}(\hat{\omega})Ae^{j\varphi}e^{j\hat{\omega}n}$$

because the frequency has changed

The new freq. is $2\hat{\omega}$

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PROBLEM 6.3:

$$y[n] = x[-n]$$

(a) $x[n] = Ae^{j\varphi}e^{+j\hat{\omega}n}$

$$y[n] = Ae^{j\varphi}e^{j\hat{\omega}(-n)} = Ae^{j\varphi}e^{-j\hat{\omega}n}$$

(b) NO.

The output cannot be written as

$$y[n] = \mathcal{H}(\hat{\omega})Ae^{j\varphi}e^{j\hat{\omega}n}$$

because the frequency has changed from $+\hat{\omega}$ to $-\hat{\omega}$.

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PROBLEM 6.4:

$$y[n] = 2x[n] - 3x[n-1] + 2x[n-2]$$

(a) $$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{2} b_k e^{-j\hat{\omega}k} = 2 - 3e^{-j\hat{\omega}} + 2e^{-j2\hat{\omega}}$$

Simplify with symmetry:

$$\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}\left\{2e^{j\hat{\omega}} - 3 + 2e^{-j\hat{\omega}}\right\}$$

$$= \underbrace{(4\cos\hat{\omega} - 3)}_{\text{THIS TERM IS MAG, EXCEPT FOR A POSSIBLE MINUS SIGN}} \underbrace{e^{-j\hat{\omega}}}_{\text{PHASE}}$$

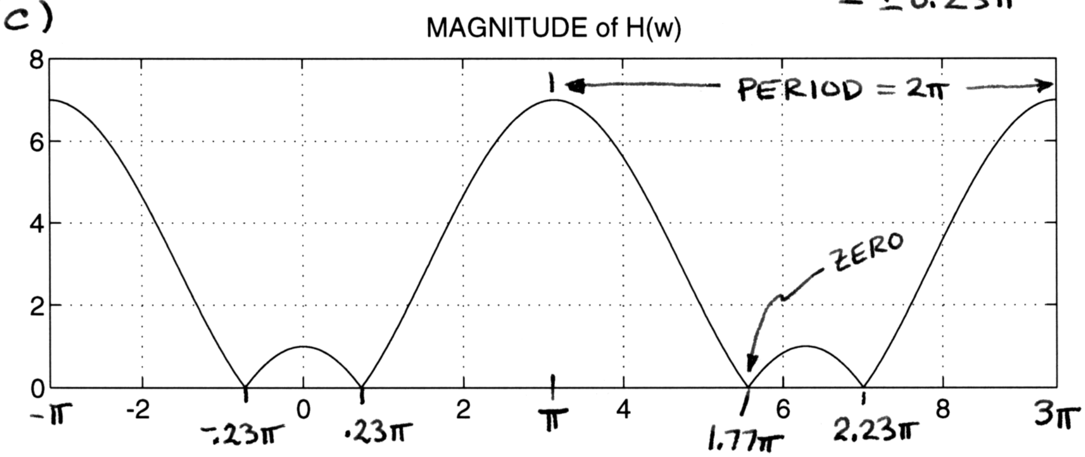
(b) $\cos\hat{\omega}$ has PERIOD $= 2\pi \Rightarrow \mathcal{H}(\hat{\omega})$ has period $= 2\pi$

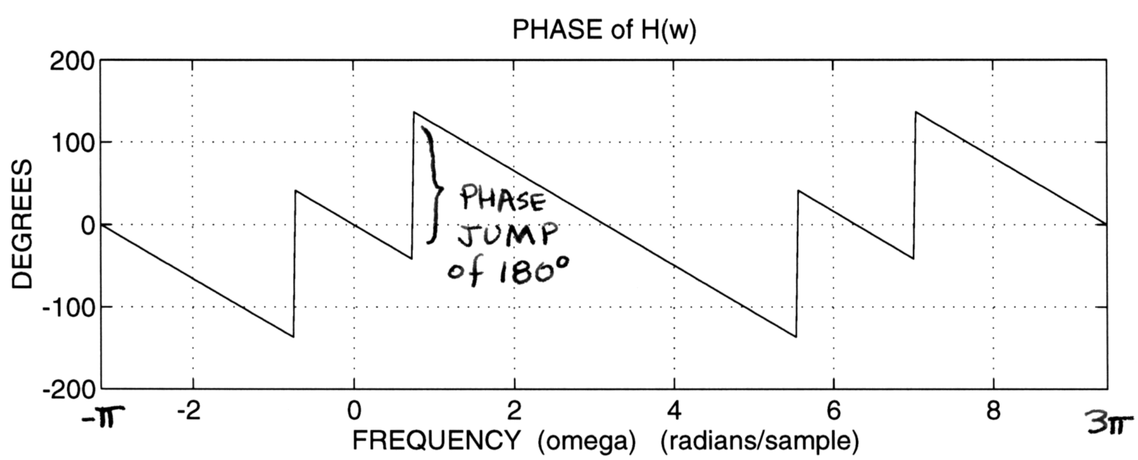
(d) OUTPUT IS ZERO WHEN $\mathcal{H}(\hat{\omega}) = 0$

$\Rightarrow$ SOLVE $4\cos\hat{\omega} - 3 = 0 \Rightarrow \hat{\omega} = \cos^{-1}(3/4)$

$= \pm 0.23\pi$

(c)





PROBLEM 6.4 (more):

(e) $\mathcal{H}(\hat{\omega})$ at $\hat{\omega} = \pi/13$ is $\mathcal{H}(\pi/13) = 0.8838 e^{-j0.077\pi}$

Since the freq. response alters mag & phase of the input, we can get output via:

$$x[n] = \sin\left(\frac{\pi}{13}n\right) = \cos\left(\frac{\pi}{13}n - \frac{\pi}{2}\right).$$

$$\Rightarrow y[n] = 0.8838 \cos\left(\frac{\pi}{13}n - 0.5\pi - 0.077\pi\right)$$

$$= 0.8838 \cos\left(\frac{\pi}{13}n - 0.577\pi\right) \quad \underline{\text{for all } n}$$

Another approach which uses linearity:

$$\sin\left(\frac{\pi}{13}n\right) = \frac{1}{2j} e^{j\pi n/13} - \frac{1}{2j} e^{-j\pi n/13}$$

$$\Rightarrow y[n] = \mathcal{H}\left(\frac{\pi}{13}\right) \frac{1}{2j} e^{j\pi n/13} - \mathcal{H}\left(-\frac{\pi}{13}\right) \frac{1}{2j} e^{-j\pi n/13}$$

$$= \frac{1}{2j}\left\{0.8838 e^{j(\pi n/13 - 0.077\pi)} - 0.8838 e^{-j(\pi n/13 - 0.077\pi)}\right\}$$

$$= 0.8838 \sin\left(\frac{\pi}{13}n - 0.077\pi\right)$$

or $y[n] = 0.8838 \cos\left(\frac{\pi}{13}n - 0.577\pi\right)$.

NOTE: THIS METHODS TRACKS THE POSITIVE AND NEGATIVE FREQUENCY COMPONENTS THROUGH THE SYSTEM SEPARATELY.

PROBLEM 6.5:

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(a) use filter coeffs: $\{b_k\} = \{1, 2, 1\}$

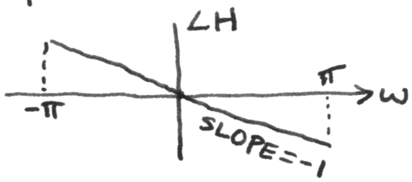
$$\mathcal{H}(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

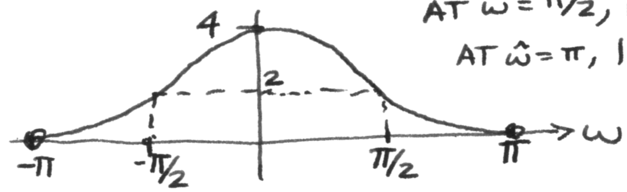
(b) $\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$

phase $= -\hat{\omega}$ MAG $= 2 + 2\cos\hat{\omega}$ $|H| = 4$ at $\hat{\omega} = 0$

AT $\hat{\omega} = \pi/2$, $|H| = 2$

AT $\hat{\omega} = \pi$, $|H| = 0$





(c) $x[n] = 10 + 4\cos(\frac{\pi}{2}n + \pi/4)$

$$= 10 + 2e^{j\pi/4}e^{j\pi/2 n} + 2e^{-j\pi/4}e^{-j\pi/2 n}$$

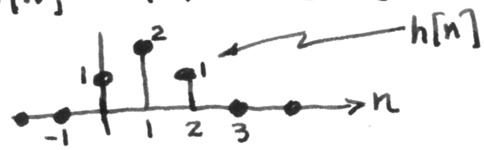
$$y[n] = 10\mathcal{H}(0) + \mathcal{H}(\pi/2)2e^{j\pi/4}e^{j\pi/2 n} + 2\mathcal{H}(-\pi/2)e^{-j\pi/4}e^{-j\frac{\pi}{2}n}$$

$$\mathcal{H}(0) = 4e^{j0} \qquad \mathcal{H}(\pi/2) = e^{-j\pi/2}(2) \qquad \mathcal{H}(-\pi/2) = 2e^{j\pi/2}$$

$$\Rightarrow y[n] = 40 + 4e^{-j\pi/2}e^{j\pi/4}e^{j\pi/2 n} + 4e^{j\pi/2}e^{-j\pi/4}e^{-j\pi/2 n}$$

$$= 40 + 8\cos(\tfrac{\pi}{2}n - \pi/4)$$

(d) $x[n] = \delta[n] \implies y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$



(e) $x[n] = u[n]$

$$y[n] = u[n] + 2u[n-1] + u[n-2]$$

$y[n] = 0$ for $n < 0$

$$y[0] = u[0] + 2u[-1] + u[-2] = 1 + 0 + 0 = 1$$

$$y[1] = u[1] + 2u[0] + u[-1] = 1 + 2 + 0 = 3$$

$$y[2] = u[2] + 2u[1] + u[0] = 1 + 2 + 1 = 4$$

$y[n] = 4$ for $n \geq 2$.

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PROBLEM 6.6:

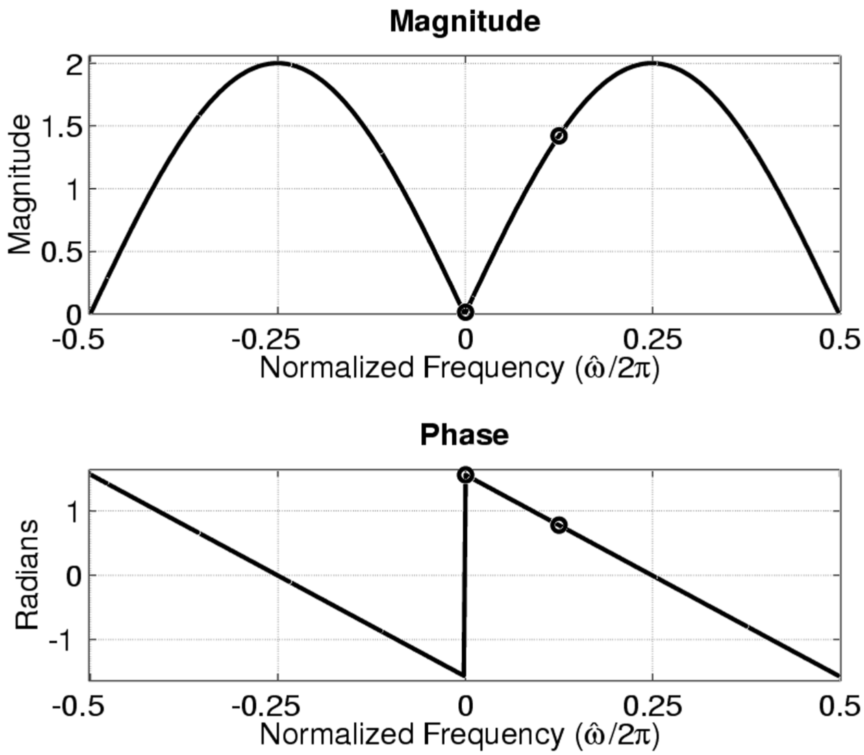
(a) $y[n] = x[n] - x[n-2]$

The filter coefficients are $\{b_k\} = \{1, 0, -1\}$

$$\mathcal{H}(\hat{\omega}) = \sum_{k=0}^{2} b_k e^{-j\hat{\omega}k} = 1 - e^{-j2\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}(e^{j\hat{\omega}} - e^{-j\hat{\omega}}) = 2e^{j\pi/2}e^{-j\hat{\omega}} \sin\hat{\omega}$$

(b) MATLAB plots



(c) $x[n] = 4 + \cos(\frac{\pi}{4}n - \frac{\pi}{4})$

Need $\mathcal{H}(0)$ Need $\mathcal{H}(\frac{\pi}{4})$

$$\mathcal{H}(0) = 2e^{j\pi/2}e^{-j0}\sin(0) = 0$$

$$\mathcal{H}(\tfrac{\pi}{4}) = 2e^{j\pi/2}e^{-j\pi/4}\sin(\pi/4) = \sqrt{2}\, e^{j\pi/4}$$

PHASE CHANGE

$$y[n] = 0 + \sqrt{2}\cos(\tfrac{\pi}{4}n - \tfrac{\pi}{4} + \tfrac{\pi}{4})$$

$$= \sqrt{2}\cos(\tfrac{\pi}{4}n) \quad \text{for } \underline{\text{all}}\ n$$

(d) $x_1[n] = x[n]u[n]$, so $x_1[n] = 0$ for $n<0$

The FIR filter is

$$y[n] = x[n] - x[n-2]$$

Once $(n-2) \geq 0$, the same numbers are involved in the calculation, so $y_1[n] = y[n]$ for $n \geq 2$. When $n<2$, then $x_1[n-2] = 0$ and $x_1[n-2] \neq x[n-2]$, so $y_1[n] \neq y[n]$ for $n<2$.

Here are the actual values for $y_1[n]$:

$$y_1[n] = 0 \quad \text{for } n<0$$

$$y_1[0] = x_1[0] - x_1[-2] = x_1[0] = 4 + \sqrt{2}/2$$

$$y_1[1] = x_1[1] - x_1[-1] = x_1[1] = 4 + 1 = 5$$

$$y_1[2] = x_1[2] - x_1[0] = 4 + \sqrt{2}/2 - (4 + \sqrt{2}/2) = 0$$

$$y_1[n] = \sqrt{2}\cos\left(\tfrac{\pi}{4}n\right) \quad \text{for } n \geq 2.$$

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PROBLEM 6.7:

(a) $H(e^{j\hat{\omega}}) = 1 + 2e^{-j3\hat{\omega}}$

Solution: Use the fact that the frequency response for $\delta[n - n_0]$ is $H(e^{j\hat{\omega}}) = e^{-j3\hat{\omega}}$.

$$h[n] = \delta[n] + 2\delta[n-3]$$

(b) $H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}}\cos(\hat{\omega})$

Solution: Use the inverse Euler formula to write the frequency response in terms of complex exponentials.

$$H(e^{j\hat{\omega}}) = 2e^{-j3\hat{\omega}}\cos(\hat{\omega}) = e^{-j3\hat{\omega}}\left(e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right)$$
$$H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} + e^{-j4\hat{\omega}}$$
$$\Rightarrow h[n] = \delta[n-2] + \delta[n-4]$$

(c) $H(e^{j\hat{\omega}}) = e^{-j4.5\hat{\omega}}\dfrac{\sin(5\hat{\omega})}{\sin(\hat{\omega}/2)}$

Solution: Use the fact that the frequency response for an L-point running sum filter is:

$$H_L(e^{j\hat{\omega}}) = e^{-j\hat{\omega}(L-1)/2}\frac{\sin(L\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

Thus, we see that $L/2 = 5$, or $L = 10$, and we can rewrite the frequency response as

$$H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}(10-1)/2}\frac{\sin(10\hat{\omega}/2)}{\sin(\hat{\omega}/2)}$$

because $(L-1)/2$ is equal to 4.5 when $L = 10$. Having made these identifications in the formula for $H(e^{j\hat{\omega}})$, we get the impulse response of the 10-point running-sum filter:

$$\begin{aligned} h[n] &= u[n] - u[n-10] \\ &= \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4]\ldots \\ &\quad + \delta[n-5] + \delta[n-6] + \delta[n-7] + \delta[n-8] + \delta[n-9] \end{aligned}$$

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PROBLEM 6.8:

(a) $$\mathcal{H}(\hat\omega) = (1+e^{-j\hat\omega})(1-\underbrace{2\cos(2\pi/3)}_{=2(1/2)=1}e^{-j\hat\omega}+e^{-j2\hat\omega})$$

$$= (1+e^{-j\hat\omega})(1-e^{-j\hat\omega}+e^{-j2\hat\omega})$$

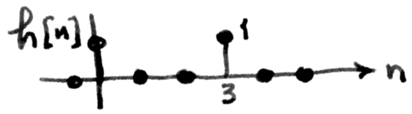
$$= 1 - e^{-j\hat\omega} + e^{-j2\hat\omega} + e^{-j\hat\omega} - e^{-j2\hat\omega} + e^{-j3\hat\omega}$$

$$= 1 + e^{-j3\hat\omega}$$

Difference Equation:

$$y[n] = x[n] + x[n-3]$$

(b) When $x[n]=\delta[n]$, $y[n] = \delta[n] + \delta[n-3]$ ← This is $h[n]$, the <u>impulse response</u>



(c) Need to find where $\mathcal{H}(\hat\omega)=0$.

$$1 + e^{-j3\hat\omega} = 0$$

$$e^{-j3\hat\omega} = -1 = e^{j\pi}e^{j2\pi\ell}$$

$$\Rightarrow e^{j\hat\omega} = e^{-j\pi/3}e^{-j2\pi\ell/3}$$

$$\Rightarrow \hat\omega = -\frac{\pi}{3} - \frac{2\pi}{3}\ell \qquad \ell = 0, 1, 2.$$

$$\hat\omega = -\frac{\pi}{3},\ -\pi,\ \text{and}\ -\frac{5\pi}{3} \leftarrow \boxed{\text{same as } +\pi/3}$$

$$y[n] = \mathcal{H}(\hat\omega)Ae^{j\varphi}e^{j\hat\omega n}$$

Thus when $\mathcal{H}(\hat\omega)=0$, the output is zero.

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PROBLEM 6.9:

(a) $\mathcal{H}(\hat{\omega}) = (1 - e^{-j\hat{\omega}})(1 - 2(0.5)\cos\frac{\pi}{6} e^{-j\hat{\omega}} + (0.5)^2 e^{-j2\hat{\omega}})$

$$= (1 - e^{-j\hat{\omega}})(1 - \tfrac{\sqrt{3}}{2} e^{-j\hat{\omega}} + \tfrac{1}{4} e^{-j2\hat{\omega}})$$

$$= 1 \underbrace{- \tfrac{1}{2}(\sqrt{3}+2)}_{-1.866} e^{-j\hat{\omega}} + \underbrace{\left(\tfrac{1}{4} + \tfrac{\sqrt{3}}{2}\right)}_{1.116} e^{-j2\hat{\omega}} - \tfrac{1}{4} e^{-j3\hat{\omega}}$$

Difference Equation:

$$y[n] = x[n] - 1.866\,x[n-1] + 1.116\,x[n-2] - \tfrac{1}{4}\,x[n-3]$$

(b) when $x[n] = \delta[n]$, $y[n] = h[n]$ <u>impulse</u> <u>response</u>

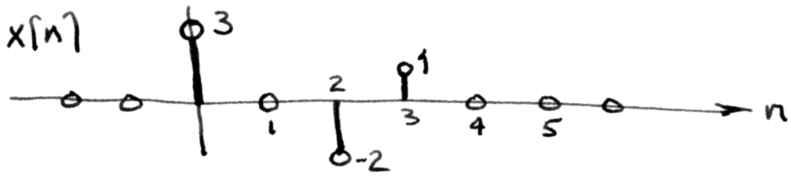
$$h[n] = \delta[n] - 1.866\,\delta[n-1] + 1.116\,\delta[n-2] - \tfrac{1}{4}\,\delta[n-3]$$

(c) Find $\hat{\omega}$ where $\mathcal{H}(\hat{\omega}) = 0$

The only frequency is $\hat{\omega} = 0$, because then the factor $(1 - e^{-j\hat{\omega}}) = 0$. The other two factors in $\mathcal{H}(\hat{\omega})$ are never zero for $-\pi \le \hat{\omega} \le \pi$.

PROBLEM 6.10:

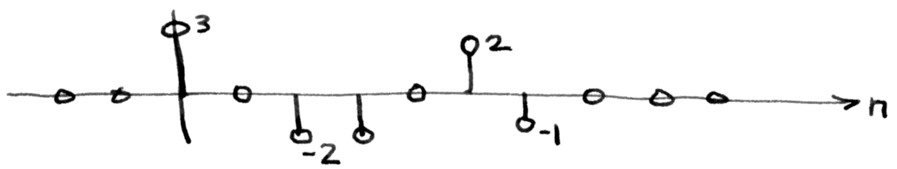
(a)



(b) Use LINEARITY & TIME-INVARIANCE

$$\left.\begin{aligned} 3\delta[n] &\longrightarrow 3\delta[n]-3\delta[n-3] \\ -2\delta[n-2] &\longrightarrow -2\delta[n-2]+2\delta[n-5] \\ \delta[n-3] &\longrightarrow \delta[n-3]-\delta[n-6] \end{aligned}\right\}\text{ Add these together}$$

$$\text{OUTPUT} = 3\delta[n]-2\delta[n-2]-2\delta[n-3]+2\delta[n-5]-\delta[n-6].$$



(c) Use the third input/output pair:

$\mathcal{H}(\pi/3) = 2$ (no phase).

$\therefore \cos(\pi(n-3)/3) \longrightarrow 2\cos(\pi(n-3)/3).$

(d) There is no direct evidence about $\mathcal{H}(\pi/2)$.

BUT use impulse response to get $\{b_k\}$.

$h[n] = \delta[n] - \delta[n-3].$

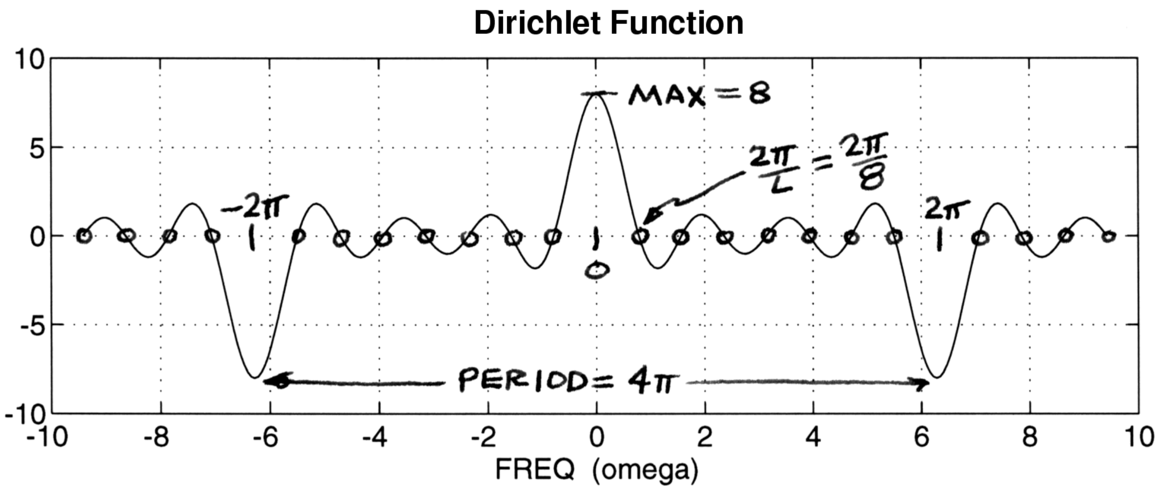
$\Rightarrow \{b_k\} = \{1, 0, 0, -1\}$

$\Rightarrow \mathcal{H}(\hat{\omega}) = 1 - e^{-j3\hat{\omega}}$

$\mathcal{H}(\pi/2) = 1 - e^{-j3\pi/2}$
$= 1 + j$

$\therefore \mathcal{H}(\pi/2) \neq 0$

PROBLEM 6.11:



(a) zero crossings at multiples of $\frac{2\pi}{L} = \frac{2\pi}{8} = \frac{\pi}{4}$

(b) PERIOD $= 4\pi$ (NOTE: PERIOD $= 2\pi$ when L is odd)

(c) MAX at $\hat{\omega} = 0, \pm 2\pi, \pm 4\pi$.

Take LIMIT

$$\lim_{\hat{\omega} \to 0} \frac{\sin 4\hat{\omega}}{\sin \hat{\omega}/2} \longrightarrow \frac{4\hat{\omega}}{\hat{\omega}/2} \longrightarrow 8$$

BECAUSE $\sin\theta \approx \theta$ when $\theta \to 0$.

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PROBLEM 6.12:

$$y[n] = x[n] - 3x[n-1] + 3x[n-2] - x[n-3]$$

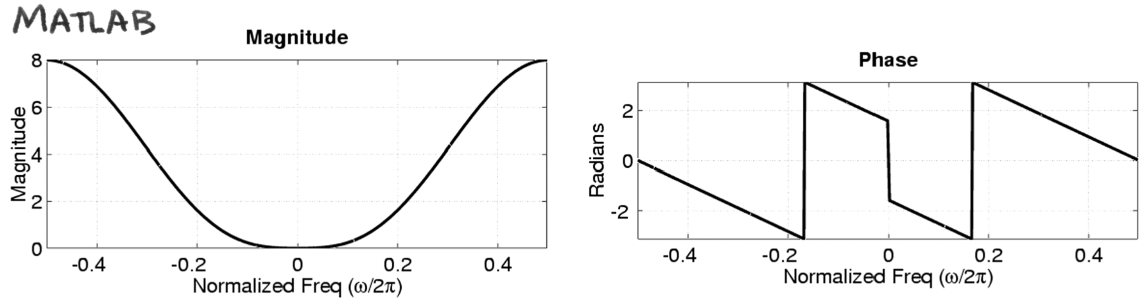
(a) use filter coeffs: $\{b_k\} = \{1, -3, 3, -1\}$

$$\mathcal{H}(\hat{\omega}) = 1 - 3e^{-j\hat{\omega}} + 3e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} = (1 - e^{-j\hat{\omega}})^3$$

$$= e^{-j3\hat{\omega}/2}\left(\frac{e^{j\hat{\omega}/2} - e^{-j\hat{\omega}/2}}{(2j)^3}\right)^3 (2j)^3 \qquad (2e^{j\pi/2})^3 = 8e^{-j\pi/2}$$

$$= 8e^{j(-\pi/2 - 3\hat{\omega}/2)} \sin^3(\hat{\omega}/2)$$

(b) MATLAB



(c) $x[n] = 10 + 4\cos(\frac{\pi}{2}n + \frac{\pi}{4})$

$$y[n] = 10\mathcal{H}(0) + 4|\mathcal{H}(\tfrac{\pi}{2})|\cos(\tfrac{\pi}{2}n + \tfrac{\pi}{4} + \angle\mathcal{H}(\tfrac{\pi}{2}))$$

$$\mathcal{H}(\pi/2) = 8e^{-j(\pi/2 + 3\pi/4)}\sin^3(\tfrac{\pi}{4})$$

$$= 8\left(\tfrac{\sqrt{2}}{2}\right)^3 e^{-j5\pi/4} = 2\sqrt{2}\, e^{-j5\pi/4}$$

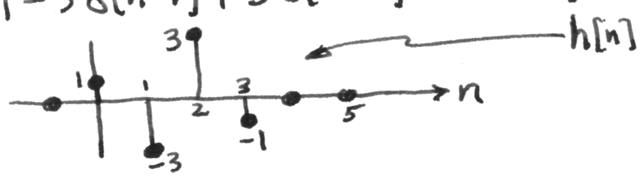
$$\mathcal{H}(0) = 0$$

$$\Rightarrow \; y[n] = 10(0) + 8\sqrt{2}\cos(\tfrac{\pi}{2}n + \tfrac{\pi}{4} - \tfrac{5\pi}{4})$$

$$= 8\sqrt{2}\cos(\tfrac{\pi}{2}n - \pi)$$

(d) $x[n] = \delta[n]$

$$y[n] = \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]$$



(e) Use superposition, so just add the results from (c) and (d)

$$y[n] = 8\sqrt{2}\cos(\tfrac{\pi}{2}n - \pi) + \delta[n] - 3\delta[n-1] + 3\delta[n-2] - \delta[n-3]$$

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PROBLEM 6.13:

(a) $y[n] = y_3[n] = x_3[n-1] + x_3[n-2]$ $x_3[n] = y_2[n]$

$$= y_2[n-1] + y_2[n-2]$$

$$= (x_2[n-1] + x_2[n-3]) + (x_2[n-2] + x_2[n-4])$$

Now replace $x_2[n]$ with $y_1[n]$

$$y[n] = y_1[n-1] + y_1[n-2] + y_1[n-3] + y_1[n-4]$$

$$= (x_1[n-1] - x_1[n-2]) + (x_1[n-2] - x_1[n-3])$$
$$+ (x_1[n-3] - x_1[n-4]) + (x_1[n-4] - x_1[n-5])$$

CANCEL

$y[n] = x_1[n-1] - x_1[n-5]$ $x_1[n] = x[n]$

$$y[n] = x[n-1] - x[n-5]$$

(b) Same thing as part (a) but use $\mathcal{H}_i(\hat{\omega})$

$\mathcal{H}_1(\hat{\omega}) = 1 - e^{-j\hat{\omega}}$

$\mathcal{H}_2(\hat{\omega}) = 1 + e^{-j2\hat{\omega}}$

$\mathcal{H}_3(\hat{\omega}) = e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$

Multiply these together

$$\mathcal{H}(\hat{\omega}) = \mathcal{H}_1(\hat{\omega})\mathcal{H}_2(\hat{\omega})\mathcal{H}_3(\hat{\omega})$$

$$= (1 - e^{-j\hat{\omega}})(1 + e^{-j2\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

$$= (1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}})(e^{-j\hat{\omega}} + e^{-j2\hat{\omega}})$$

$$= e^{-j\hat{\omega}} - e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}} - e^{-j4\hat{\omega}}$$
$$+ e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\Rightarrow \quad y[n] = x[n-1] - x[n-5]$$

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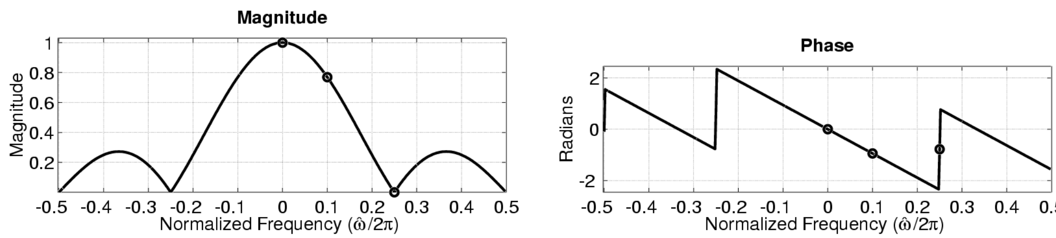
PROBLEM 6.14:

(a) $h[n] = \frac{1}{4}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{4}\delta[n-2] + \frac{1}{4}\delta[n-3]$

(b)
$$\begin{aligned}\mathcal{H}(\hat\omega) &= \frac{1}{4}\left(1 + e^{-j\hat\omega} + e^{-j2\hat\omega} + e^{-j3\hat\omega}\right)\\ &= \frac{1}{4}e^{-j\frac{3}{2}\hat\omega}\left(e^{j\frac{3}{2}\hat\omega} + e^{j\frac{1}{2}\hat\omega} + e^{-j\frac{1}{2}\hat\omega} + e^{-j\frac{3}{2}\hat\omega}\right)\\ &= \frac{1}{2}e^{-j\frac{3}{2}\hat\omega}\left(\cos(\tfrac{3}{2}\hat\omega) + \cos(\tfrac{1}{2}\hat\omega)\right)\end{aligned}$$

(c) MATLAB



(d) $x[n] = 5 + 4\cos(0.2\pi n) + 3\cos(0.5\pi n + \pi/4)$

Need $\mathcal{H}(0)$ Need $\mathcal{H}(0.2\pi)$ $\mathcal{H}(0.5\pi)$

$\mathcal{H}(0) = 1$

$\mathcal{H}(0.2\pi) = 0.769\, e^{-j0.3\pi}$ ANGLE $= -54° = -0.942$ rad

$\mathcal{H}(0.5\pi) = 0$

$$\Rightarrow \quad y[n] = 5 + \underbrace{4(0.769)}_{3.078}\cos(0.2\pi n - 0.3\pi)$$

(e) $x_1[n] = 0$ for $n < 0$

$$y_1[n] = \frac{1}{4}\left(x_1[n] + x_1[n-1] + x_1[n-2] + x_1[n-3]\right)$$

Since $x_1[n] = x[n]$ for $n \geq 0$, the filtered outputs will be the same when $n - 3 \geq 0$

$\Rightarrow n \geq 3$ is the region where $y_1[n] = y[n]$

Here's a table of the first few values:

n	-1	0	1	2	3	4 ...
$y[n]$	4.029	6.809	7.927	7.927	6.809	5 ...
$y_1[n]$	0	2.780	4.309	5.338	6.809	5 ...

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PROBLEM 6.15:

$$x(t) = 10 + 8\cos(200\pi t) + 6\cos(500\pi t + \pi/4)$$

$$f_s = 1000$$

$$x[n] = x(t)\Big|_{t = n/f_s = n/1000}$$

$$= 10 + 8\cos\left(200\frac{\pi n}{1000}\right) + 6\cos\left(500\frac{\pi n}{1000} + \pi/4\right)$$

$$= 10 + 8\cos(0.2\pi n) + 6\cos(0.5\pi n + \pi/4)$$

Need $\mathcal{H}(0)$ Need $\mathcal{H}(0.2\pi)$ $\mathcal{H}(0.5\pi) = 0$

Use frequency response values from Prob. 6.14

$\mathcal{H}(0) = 1$ $\mathcal{H}(0.2\pi) = 0.769\, e^{-j0.3\pi}$

$$y[n] = 10 + (0.769)8\cos(0.2\pi n - 0.3\pi)$$

$$= 10 + 6.156\cos(0.2\pi n - 0.3\pi)$$

$$y(t) = y[n]\Big|_{n \leftarrow f_s t = 1000t}$$

$$= 10 + 6.156\cos(200\pi t - 0.3\pi)$$

PROBLEM 6.16:

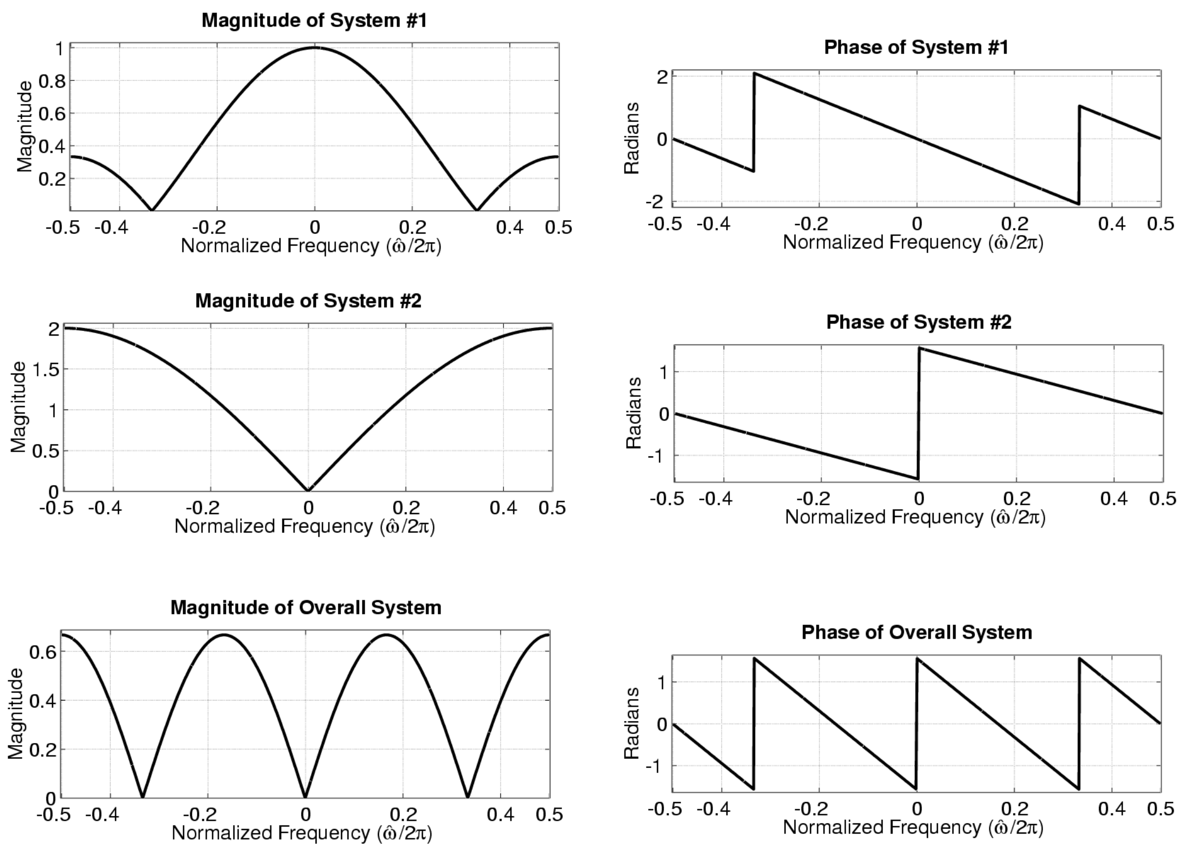
(a) System 2 will block the DC component which is a constant. $\mathcal{H}_2(0)=0$. Also $y[n]=v[n]-v[n-1]$ so the differencing operator removes DC.

(b) $\mathcal{H}(\hat{\omega})=\mathcal{H}_1(\hat{\omega})\,\mathcal{H}_2(\hat{\omega})$

$$\mathcal{H}_1(\hat{\omega})=\tfrac{1}{3}+\tfrac{1}{3}e^{-j\hat{\omega}}+\tfrac{1}{3}e^{-j2\hat{\omega}} \qquad \mathcal{H}_2(\hat{\omega})=1-e^{-j\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega})=\tfrac{1}{3}-\tfrac{1}{3}e^{-j3\hat{\omega}}$$

(c) MATLAB



(d) $y[n]=\tfrac{1}{3}x[n]-\tfrac{1}{3}x[n-3]$

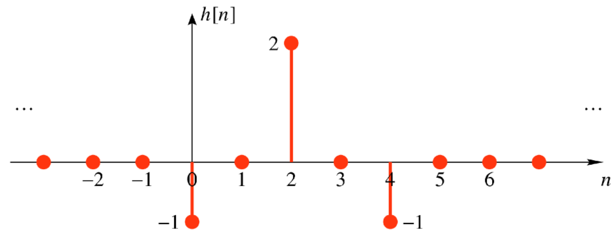
PROBLEM 6.17:

A linear time-invariant system is described by the difference equation

$$y[n] = -x[n] + 2x[n-2] - x[n-4]$$

(a) Find the impulse response $h[n]$ and plot it.

Solution:
Let $x[n]$ be $\delta[n]$, and find the output:
$h[n] = -\delta[n] + 2\delta[n-2] - \delta[n-4]$



(b) Determine an equation for the frequency response $H(e^{j\hat{\omega}})$ and express it as $H(e^{j\hat{\omega}}) = e^{-j\hat{\omega}n_0}R(e^{j\hat{\omega}})$, where $R(e^{j\hat{\omega}})$ is a real function and n_0 is an integer.

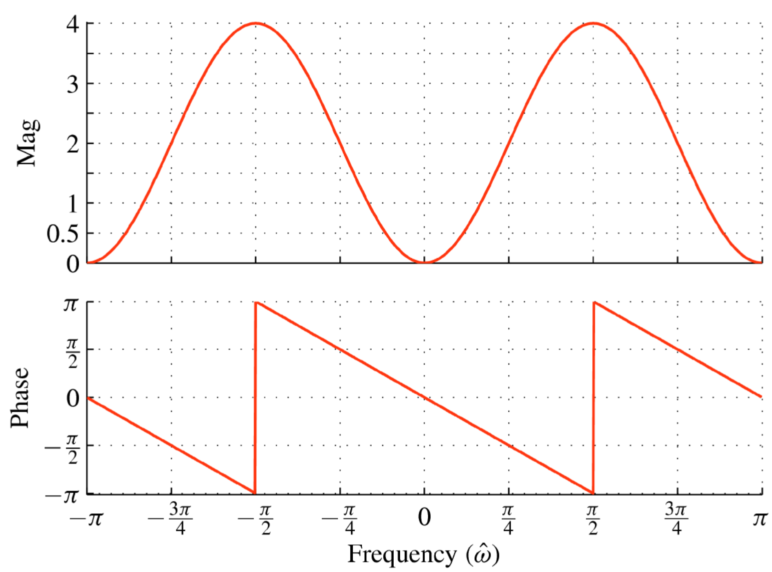
Solution: Plug into the frequency response formula:

$$\begin{aligned} H(e^{j\hat{\omega}}) &= -e^{j0} + 2e^{-j2\hat{\omega}} - e^{-j4\hat{\omega}} \\ &= e^{-j2\hat{\omega}}\left(-e^{j2\hat{\omega}} + 2 - e^{-j2\hat{\omega}}\right) \\ &= e^{-j2\hat{\omega}}\left(2 - 2\cos(2\hat{\omega})\right) = e^{-j\hat{\omega}n_0}R(e^{j\hat{\omega}}) \end{aligned}$$

Thus, $n_0 = 2$ and $R(e^{j\hat{\omega}}) = 2 - 2\cos(2\hat{\omega})$.

(c) Carefully sketch and label a plot of $|H(e^{j\hat{\omega}})|$ for $-\pi < \hat{\omega} < \pi$.

Solution: Since $R(e^{j\hat{\omega}}) \geq 0$, the magnitude is $|H(e^{j\hat{\omega}})| = R(e^{j\hat{\omega}}) = 2 - 2\cos(2\hat{\omega})$.



(d) Carefully sketch and label a plot of the principal value of the $\angle H(e^{j\hat{\omega}})$ for $-\pi < \hat{\omega} < \pi$.

Solution: The phase is $\angle H(e^{j\hat{\omega}}) = -2\hat{\omega}$, but the principal value wraps at $\hat{\omega} = \pi/2$ when $\angle H(e^{j\hat{\omega}})$ is outside of the range $[-\pi, \pi]$.

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PROBLEM 6.18:

$$x[n] = 5 + 20\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) + 10\delta[n-3]$$

Need $\mathcal{H}(0)$ — DEPENDS on $\mathcal{H}(\pi/2)$ — Need impulse response $h[n]$

$$\mathcal{H}(0) = (1 - j)(1-(-j))(1+1)$$
$$= (1-j)(1+j)2 = 2 \cdot 2 = 4$$

$$\mathcal{H}(\pi/2) = (1 - je^{-j\pi/2})(1+je^{-j\pi/2})(1+e^{-j\pi/2})$$
$$= (1-j(-j))(1+j(-j))(1-j)$$
$$= (1-1)(1+1)(1-j) = 0$$

To find $h[n]$, multiply out $\mathcal{H}(\hat{\omega})$

$$\mathcal{H}(\hat{\omega}) = (1 - je^{-j\hat{\omega}} + je^{-j\hat{\omega}} + e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}})$$
$$= (1+e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}})$$
$$= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$\Rightarrow h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

Finally,

$$y[n] = 5(4) + 0 + 10h[n-3]$$
$$= 20 + 10\delta[n-3] + 10\delta[n-4] + 10\delta[n-5] + 10\delta[n-6]$$

PROBLEM 6.19:

(a) $\mathcal{H}(\hat{\omega}) = \mathcal{H}_1(\hat{\omega})\mathcal{H}_2(\hat{\omega})$

$$\mathcal{H}_2(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$\mathcal{H}_1(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

Multiply:

$$\mathcal{H}(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + 2e^{-j\hat{\omega}} - 2e^{-j2\hat{\omega}}$$
$$+ 2e^{-j3\hat{\omega}} - 2e^{-j4\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} - e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

MANY TERMS CANCEL OUT

(b) $h[n] = \delta[n] + \delta[n-1] - \delta[n-4] - \delta[n-5]$

(c) The polynomial coefficients of $\mathcal{H}(\hat{\omega})$ define $\{b_k\}$ as $\{1, 1, 0, 0, -1, -1\}$. Use $\{b_k\}$ as filter coefficients:

$$y[n] = x[n] + x[n-1] - x[n-4] - x[n-5]$$

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PROBLEM 6.20:

(a) The highest frequency in $x(t)$ is $\omega_0 = 2\pi(500)$

To avoid aliasing we must sample at $f_s > 2f_{MAX}$

$\Rightarrow \quad f_s > 2(500\text{Hz}) = 1000$ samples/sec.

(b) $h[n] = \delta[n-10]$, f_s and ω_0 to be determined

$$x[n] = 10 + 20\cos(\omega_0 n/f_s + \pi/3)$$

$$y[n] = x[n-10] = 10 + 20\cos\left(\omega_0 \frac{(n-10)}{f_s} + \pi/3\right)$$

$$y(t) = y[n]\Big|_{n \leftarrow f_s t} = 10 + 20\cos\left(\frac{\omega_0}{f_s}(f_s t - 10) + \pi/3\right)$$

Since we want $y(t) = x(t-0.001)$, we need

$$\frac{\omega_0}{f_s}(10) = (0.001)\omega_0$$

$$\Rightarrow \quad 10/f_s = 1/1000 \quad \Rightarrow \quad f_s = 10{,}000\text{ Hz}$$

In order for the output frequency to be the same as the input frequency ω_0, there must be no aliasing. $\Rightarrow \quad 2\omega_0 < 2\pi f_s$

$$\Rightarrow \omega_0 < 2\pi(500)\text{ rad/sec}$$

(c) To have $y(t) = A$, we need $y[n] = $ constant.

Since $x[n] = 10 + 20\cos(\omega_0 n/f_s + \pi/3)$ $\quad f_s = 2000$

the filter must "null out" the cosine term

$\Rightarrow \quad \frac{\omega_0}{f_s} = \hat{\omega}_{NULL}$ where $\hat{\omega}_{NULL}$ is one of the zeros of $\mathcal{H}(\hat{\omega})$

$$\mathcal{H}(\hat{\omega}) = 0 \text{ when } \hat{\omega} = 2\pi/5,\ 4\pi/5,\ -2\pi/5,\ -4\pi/5$$

$$\therefore \quad \omega_0 = f_s\hat{\omega}_{NULL} = \{2\pi(400),\ 2\pi(800),\ -2\pi(400),\ -2\pi(800)\}$$

We must include all aliases:

$2\pi(400),\ 2\pi(2400),\ 2\pi(4400), \ldots \qquad 2\pi(400 + 2000\ell)$

$2\pi(800),\ 2\pi(2800),\ 2\pi(4800), \ldots \qquad 2\pi(800 + 2000\ell)$

$2\pi(-400),\ 2\pi(1600),\ 2\pi(3600), \ldots \qquad 2\pi(-400 + 2000\ell)$

$2\pi(-800),\ 2\pi(1200),\ 2\pi(3200), \ldots \qquad 2\pi(-800 + 2000\ell)$

PROBLEM 6.21:

(a) $x[n] = 10 + 10\cos(0.2\pi n) + 10\cos(0.5\pi n)$

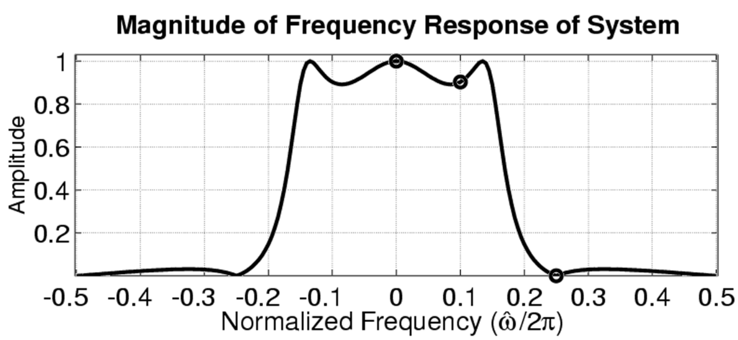
Need $\mathcal{H}(0)$ Need $\mathcal{H}(0.2\pi)$ $\mathcal{H}(0.5\pi)$

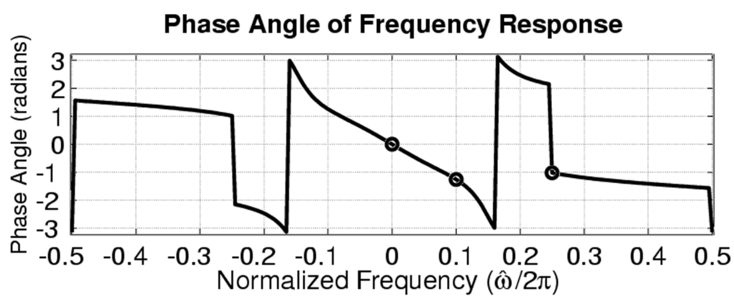
$\mathcal{H}(0) = 1$

$\mathcal{H}(0.2\pi) = 0.9027\, e^{-j0.4\pi}$ ANGLE $= -71.98^\circ = -1.26$ rads

$\mathcal{H}(0.5\pi) = 0.00089\, e^{-j0.323\pi}$ ANGLE $= -58.22^\circ = -1.02$ rads

$$y[n] = 10 + 9.027\cos(0.2\pi n - 0.4\pi) + \underbrace{(0.00089)10\cos(0.5\pi n - 0.323\pi)}_{\text{very close to zero}}$$





(b) The discontinuity at $\hat{\omega} = 2\pi(0.25)$ is caused by the zero near $\hat{\omega} = 2\pi(0.25)$. There is a sign change in $\mathcal{H}(\hat{\omega})$ which means the phase changes by π

The discontinuity at $\hat{\omega} = 2\pi(0.17)$ is a "2π-jump" which happens when the principal value of the phase tries to cross π. The arctangent calculation flips the value from π to $-\pi$ creating a "2π jump."

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