Theorem 2.8. Assume

a) $g'(p) = 0$

b) g'' is continuous on $I \ni p$ (I-interval)

Then there exists $\delta > 0$ such that for $p_0 \in [p-\delta, p+\delta]$ the sequence $\{p_n\}_{n=1}^{\infty}$

$$p_n = g(p_{n-1}) \quad n \geq 1$$

converges at least quadratically.

Consider Newton's method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

Then

$$g(x) = x - \frac{f(x)}{f'(x)}$$

Then,

$$g'(x) = 1 - \frac{[f'(x)]^2 - f(x)f''(x)}{[f'(x)]^2} = \frac{f(x)f''(x)}{[f'(x)]^2}$$

Thus, $g'(p) = 0$. Theorem 2.8 implies that Newton's method converges quadratically.

Exception: If $f'(p) = 0$ the convergence may not be quadratic.