

$L_{n,k}(x)$ for each $k=0, 1, \dots, n$
with properties

$$L_{n,k}(x_i) = 0 \quad \text{if } i \neq k$$

$$L_{n,k}(x_k) = 1$$

A polynomial of degree n which is zero
at the points

$$x_0, x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_n$$

is

$$(x-x_0)(x-x_1)\dots(x-x_{k-1})(x-x_{k+1})\dots(x-x_n)$$

To satisfy $L_{n,k}(x_k) = 1$ we must divide by
the value of this polynomial at $x = x_k$
Thus,

basis polynomial $\rightarrow L_{n,k}(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{k-1})(x-x_{k+1})\dots(x-x_n)}{(x_k-x_0)(x_k-x_1)\dots(x_k-x_{k-1})(x_k-x_{k+1})\dots(x_k-x_n)}$

Let $P(x)$ be a polynomial of degree n
satisfying

$$P(x_k) = f_k \quad k=0, 1, \dots, n$$

then

$$P(x) = f_0 L_{n,0}(x) + \dots + f_n L_{n,n}(x) = \sum_{k=0}^n f_k L_{n,k}(x)$$