PROBLEM 4.2:

$$x(t) = 7\sin(11\pi t) = 7\cos(11\pi t - \pi/2) \longrightarrow \boxed{A/D}_{f_s} \longrightarrow x[n] = A\cos(\hat{\omega}_0 n + \varphi).$$

(a) $f_s = 10$ samples/sec.

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{f_s}\right) = 7\cos\left(\frac{11\pi n}{10} - \pi/2\right).$$

$$= 7\cos\left(\frac{11\pi n}{10} - 2\pi n - \pi/2\right).$$

$$= 7\cos\left(-\frac{9\pi n}{10} - \pi/2\right) = 7\cos(.9\pi n + \pi/2).$$

$$\boxed{A = 7,\ \hat{\omega}_0 = 0.9\pi,\ \varphi = \pi/2}$$

(b) $f_s = 5$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{5}\right) = 7\cos\left(\frac{11\pi n}{5} - \pi/2\right)$$

$$= 7\cos\left(\frac{\pi n}{5} - \frac{\pi}{2}\right)$$

$$\boxed{A = 7,\ \hat{\omega}_0 = \frac{\pi}{5},\ \varphi = -\frac{\pi}{2}}$$

(c) $f_s = 15$ samples/sec

$$x(t)\Big|_{t=n/f_s} = x\left(\frac{n}{15}\right) = 7\cos\left(\frac{11\pi n}{15} - \frac{\pi}{2}\right)$$

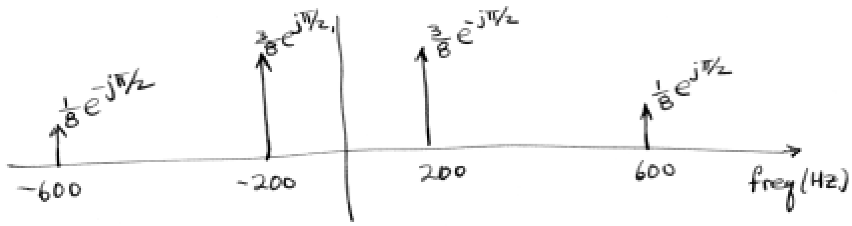
$$A = 7,\ \hat{\omega}_0 = \frac{11\pi}{15} = 2\pi\left(\frac{5.5}{15}\right)\ \&\ \varphi = -\pi/2$$

PROBLEM 4.9:

(a) Draw a sketch of the spectrum of $x(t)$ which is "sine-cubed" $x(t) = \sin^3(400\pi t)$

$$x(t) = \left(\frac{e^{j400\pi t} - e^{-j400\pi t}}{2j} \right)^3$$

$$= \frac{1}{-8j} \left\{ e^{j1200\pi t} - 3e^{j400\pi t} + 3e^{-j400\pi t} - e^{-j1200\pi t} \right).$$



$$\frac{1}{-8j} = \frac{1}{8} e^{j\pi/2}$$

(b) Determine the minimum sampling rate that can be used to sample $x(t)$ without any aliasing.

$$f_s \geq 2 f_{HIGH}$$

$$\Rightarrow f_s \geq 1200 \text{ Hz}$$

PROBLEM 11.2:

(a) The Fourier Transform (FT) of $\delta(t)$ is 1. Thus, the FT of $\delta(t-t_d)$ is $1e^{-j\omega t_d}$

using linearity of the FT.

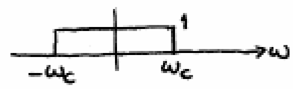
$$FT\{x(t)\} = FT\{\delta(t+1)\} + FT\{2\delta(t)\} + FT\{\delta(t-1)\}$$

$$X(j\omega) = e^{j\omega} + 2 + e^{-j\omega}$$

$$= 2 + 2\cos\omega$$ if you simplify

(b) $\dfrac{\sin(100\pi(t-2))}{\pi(t-2)}$ is a shifted "sinc" function

The FT of $\dfrac{\sin(\omega_c t)}{\pi t}$ is a rectangle



This F.T. can be found in Table 12.1.

It can also be written in terms of unit steps as $u(\omega+\omega_c) - u(\omega-\omega_c)$. In this case, $\omega_c = 100\pi$ rad/s

Using the shift property with $t_d = 2$

$$X(j\omega) = \{u(\omega+100\pi) - u(\omega-100\pi)\}e^{-j2\omega}$$

(c) The F.T. of $e^{-at}u(t)$ is $\frac{1}{a+j\omega}$.

$$x(t) = e^{-t}u(t) - \underbrace{e^{-t}u(t-4)}_{\text{This is NOT a pure shift}} = e^{-t}u(t) - e^{-4}\underbrace{e^{-(t-4)}u(t-4)}_{\text{This re-write shows the shift of both terms}}$$

$$X(j\omega) = \frac{1}{1+j\omega} - e^{-4}\frac{e^{-j4\omega}}{1+j\omega}$$

$$= \frac{1-e^{-4(1+j\omega)}}{1+j\omega}$$

PROBLEM 11.3:

The general approach is to use Tables plus some algebraic manipulations:

(a) $\dfrac{j\omega}{0.1+j\omega}e^{-j0.2\omega} = X_1(j\omega)e^{-j0.2\omega}$ ← use time shifting

If $X_1(j\omega) = \dfrac{j\omega}{0.1+j\omega}$, then $X_1(j\omega) = j\omega X_2(j\omega)$ ← use derivative

If $X_2(j\omega) = \dfrac{1}{0.1+j\omega} \Rightarrow x_2(t) = e^{-0.1t}u(t)$

$\Rightarrow x_1(t) = \dfrac{d}{dt}x_2(t) = e^{-0.1t}\delta(t) - 0.1e^{-0.1t}u(t) = \delta(t) - 0.1e^{-0.1t}u(t)$

$x(t) = x_1(t-0.2) = \delta(t-0.2) - 0.1e^{-0.1(t-0.2)}u(t-0.2)$

(b) $X(j\omega) = 2 + 2\cos\omega = 2 + e^{-j\omega} + e^{j\omega}$ ← use shifting

$x(t) = 2\delta(t) + \delta(t-1) + \delta(t+1)$

(c) Use Table entry $\dfrac{1}{a+j\omega} \longrightarrow e^{-at}u(t)$

$x(t) = e^{-t}u(t) - e^{-2t}u(t)$

(d) use Table entry: $2\pi\delta(\omega-\omega_0) \longrightarrow e^{j\omega_0 t}$

$X(j\omega) = j\frac{2\pi}{2\pi}\delta(\omega-100\pi) - j\frac{2\pi}{2\pi}\delta(\omega-(-100\pi))$

$x(t) = \dfrac{j}{2\pi}e^{j100\pi t} - \dfrac{j}{2\pi}e^{-j100\pi t}$

$= \dfrac{-1}{\pi}\left\{\dfrac{1}{2j}e^{j100\pi t} - \dfrac{1}{2j}e^{-j100\pi t}\right\}$ ← use Inverse Euler

$x(t) = -\dfrac{1}{\pi}\sin(100\pi t)$

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PROBLEM 11.5:

Use the derivative property:

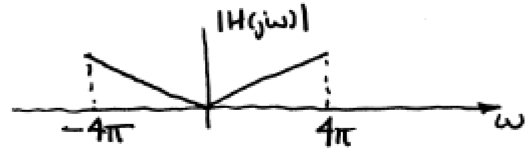
$$\frac{d}{dt}x(t) \xrightarrow{F.T.} j\omega X(j\omega)$$

Here, $x(t) = \dfrac{\sin(4\pi t)}{\pi t}$

$$\Rightarrow X(j\omega) = u(\omega+4\pi) - u(\omega-4\pi)$$

Then, $H(j\omega) = j\omega\left[u(\omega+4\pi) - u(\omega-4\pi)\right]$

$$|H(j\omega)| = |\omega| \quad \text{for } -4\pi < \omega < 4\pi$$



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