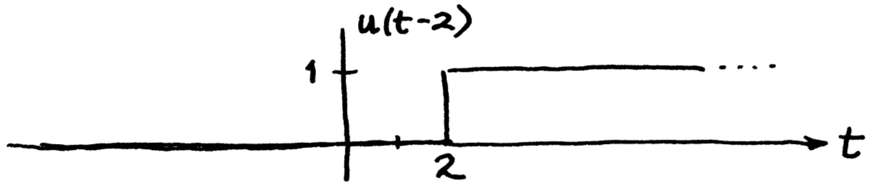
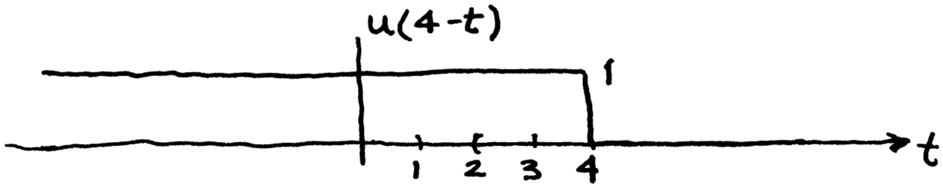
PROBLEM 9.1:

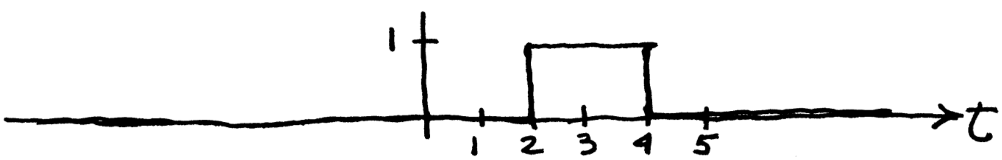
(a) $u(t-2)$ is one when $t-2>0 \Rightarrow t>2$



(b) $u(4-t)$ is one when $4-t>0 \Rightarrow t<4$



(c) Plot of $u(t-2)u(4-t)$ is one for $2<t<4$



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PROBLEM 9.2:

(a) An *exponentiation system* is defined by the input/output relation $y(t) = \exp\{x(t+2)\} = e^{x(t+2)}$

(i) *Linear:* The system is ***not*** linear because the sum of two inputs will give an output that is the product of the corresponding outputs:

$$\begin{aligned} x_1(t) &\rightarrow y_1(t) = e^{x_1(t+2)} \\ x_2(t) &\rightarrow y_2(t) = e^{x_2(t+2)} \\ x_1(t) + x_2(t) &\rightarrow e^{x_1(t+2)+x_2(t+2)} = e^{x_1(t+2)} e^{x_2(t+2)} = y_1(t) y_2(t) \end{aligned}$$

(ii) *Time-invariant:* The system is time-invariant because the system definition is a point-wise operator:

$$\begin{aligned} x_1(t) &\rightarrow y_1(t) = e^{x_1(t+2)} \\ x_1(t-t_1) &\rightarrow y_2(t) = e^{x_1(t+2-t_1)} = e^{x_1((t-t_1)+2)} = y_1(t-t_1) \end{aligned}$$

(iii) *Stable:* The system is stable because the system definition is a point-wise operator. If the input signal is bounded by M_x, i.e., $\max\{|x[n]|\} < M_x$, then the output signal is bounded by $M_y = e^{M_x}$.

(iv) *Causal:* The system is ***not*** causal because the system definition involves a time-shift of $(t+2)$ which is a shift by -2. Here is a counter-example:

$$x_1(t) = u(t) \rightarrow y_1(t) = e^{u(t+2)} = e^1 u(t+2)$$

In other words, the input "starts" at $t = 0$, while the output "starts earlier" at $t = -2$.

(b) A *phase modulator* is a system whose input and output satisfy a relation of the form $y(t) = \cos[\omega_c t + x(t)]$

(i) *Linear:* The system is ***not*** linear because the sum of two inputs will give an output that is the not the sum of the corresponding outputs. Let one of the input signals be the zero signal to get a counterexample:

$$\begin{aligned} x_1(t) &\rightarrow y_1(t) = \cos[\omega_c t + x_1(t)] \\ x_2(t) = 0 &\rightarrow y_2(t) = \cos[\omega_c t + x_2(t)] = \cos[\omega_c t] \\ x_1(t) + x_2(t) &\rightarrow \cos[\omega_c t + x_1(t) + x_2(t)] = \cos[\omega_c t + x_1(t)] = y_1(t) \neq y_1(t) + y_2(t) \end{aligned}$$

(ii) *Time-invariant:* The system is ***not*** time-invariant because the system definition contains a component that does not depend on $x(t)$. Here is a counterexample with a unit-step signal:

$$\begin{aligned} x_1(t) = \pi u(t) &\rightarrow y_1(t) = \cos[\omega_c t + \pi u(t)] = \cos[\omega_c t] u(-t) - \cos[\omega_c t] u(t) \\ x_1(t-1) = \pi u(t-1) &\rightarrow y_2(t) = \cos[\omega_c t + \pi u(t-1)] = \cos[\omega_c t] u(1-t) - \cos[\omega_c t] u(t-1) \\ \text{but, } y_1(t-1) &= \cos[\omega_c (t-1)] u(1-t) - \cos[\omega_c (t-1)] u(t-1) \end{aligned}$$

Thus, $y_2(t) \neq y_1(t-1)$ which means that $y_2(t)$ is not a shifted version of $y_1(t)$.

PROBLEM 9.2 (more):

(iii) *Stable:* The system is stable because the output will always be bounded by one, independent of the values of $x(t)$.

(iv) *Causal:* The system is causal because the output $y(t)$ depends only on the value of the input $x(t)$ **at the same time**. No values of $x(t)$ from the future (or the past) are used.

(c) An *amplitude modulator* is a system whose input and output satisfy a relation of the form $y(t) = [A + x(t)]\cos(\omega_c t)$

(i) *Linear:* The system is ***not*** linear because the sum of two inputs will give an output that is the not the sum of the corresponding outputs. Let one of the input signals be the zero signal to get a counterexample:

$$\begin{aligned} x_1(t) &\to y_1(t) = [A + x_1(t)]\cos(\omega_c t) \\ x_2(t) = 0 &\to y_2(t) = [A + x_2(t)]\cos(\omega_c t) = A\cos[\omega_c t] \\ x_1(t) + x_2(t) &\to [A + x_1(t) + x_2(t)]\cos(\omega_c t) = [A + x_1(t)]\cos(\omega_c t) = y_1(t) \neq y_1(t) + y_2(t) \end{aligned}$$

(ii) *Time-invariant:* The system is ***not*** time-invariant because the system definition contains a component that does not depend on $x(t)$. Here is a counterexample with a unit-step signal:

$$\begin{aligned} x_1(t) = -Au(t) &\to y_1(t) = [A - Au(t)]\cos(\omega_c t) = A\cos[\omega_c t]u(-t) \\ x_1(t-1) = Au(t-1) &\to y_2(t) = [A - Au(t-1)]\cos(\omega_c t) = A\cos[\omega_c t]u(1-t) \\ \text{but, } y_1(t-1) &= A\cos[\omega_c(t-1)]u(1-t) \end{aligned}$$

Thus, $y_2(t) \neq y_1(t-1)$ which means that $y_2(t)$ is not a shifted version of $y_1(t)$.

(iii) *Stable:* The system is stable because the output will always be bounded by $|A + \max\{|x[n]|\}|$.

(iv) *Causal:* The system is causal because the output $y(t)$ depends only on the value of the input $x(t)$ **at the same time**. No values of $x(t)$ from the future (or the past) are used.

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PROBLEM 9.2 (more):

(d) A system that takes the even part of an input signal is defined by a relation of the form $y(t) = \mathcal{E}v\{x(t)\} = \dfrac{x(t) + x(-t)}{2}$

(i) *Linear:* The system is linear, so we have to prove both the scaling property and the superposition property:

$$\begin{aligned}
x_1(t) &\to y_1(t) = \tfrac{1}{2}x_1(t) + \tfrac{1}{2}x_1(-t) \\
x_2(t) &\to y_2(t) = \tfrac{1}{2}x_2(t) + \tfrac{1}{2}x_2(-t) \\
x_1(t) + x_2(t) &\to \tfrac{1}{2}(x_1(t) + x_2(t)) + \tfrac{1}{2}(x_1(-t) + x_2(-t)) \\
&\qquad = \tfrac{1}{2}(x_1(t) + x_1(-t)) + \tfrac{1}{2}(x_2(t) + x_2(-t)) = y_1(t) + y_2(t) \\
\beta x_1(t) &\to \tfrac{1}{2}(\beta x_1(t)) + \tfrac{1}{2}(\beta x_1(-t)) = \beta\left(\tfrac{1}{2}x_1(t) + \tfrac{1}{2}x_1(-t)\right) = \beta y_1(t)
\end{aligned}$$

(ii) *Time-invariant:* The system is ***not*** time-invariant because the system definition contains a flip. Here is a counterexample with a unit-impulse signal:

$$\begin{aligned}
x_1(t) = \delta(t) &\to y_1(t) = \tfrac{1}{2}\{\delta(t) + \delta(-t)\} = \delta(t) \\
x_1(t-1) = \delta(t-1) &\to y_1(t) = \tfrac{1}{2}\{\delta(t-1) + \delta(-t-1))\} = \tfrac{1}{2}
\end{aligned}$$

$$delta(t-1) + \tfrac{1}{2}\delta(t+1)\}$$

$$\text{but, } y_1(t-1) = \delta(t-1)$$

Thus, $y_2(t) \neq y_1(t-1)$ which means that $y_2(t)$ is not a shifted version of $y_1(t)$.

(iii) *Stable:* The system is stable because the output will always be bounded by $\max\{|x[n]|\}|$.

$$\max\{|y[n]|\} = \max\{|\tfrac{1}{2}x(t) + \tfrac{1}{2}x(-t)|\} \leq \tfrac{1}{2}\max\{|x[n]|\}| + \tfrac{1}{2}\max\{|x[n]|\}|$$

(iv) *Causal:* The system is ***not*** causal because the flip component of the system definition creates a component in negative time. The signal $\delta(t-1)$ provides a counterexample. From above, the input "starts" at $t = 1$, while the output "starts earlier" at $t = -1$.

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PROBLEM 9.3:

(a) $\delta(t-10) * [\delta(t+10) + 2e^{-t}u(t) + \cos(100\pi t)]$

$= \delta(t-10) * \delta(t+10) + \delta(t-10) * 2e^{-t}u(t) + \delta(t-10) * \cos(100\pi t)$

$= \delta(t) \quad + 2e^{-(t-10)}u(t-10) + \cos[100\pi(t-10)]$

(b) $\cos(100\pi t)[\delta(t) + \delta(t-.002)]$

$= \cos(100\pi \cdot 0)\delta(t) + \cos(100\pi(.002))\delta(t-.002)$

$= 1\,\delta(t) + \cos(0.2\pi)\delta(t-.002) = \delta(t) + 0.809\,\delta(t-.002)$

(c) $\frac{d}{dt}\left[e^{-2(t-2)}u(t-2)\right]$ Use formula for derivative of a product.

$= \frac{d}{dt}\left[e^{-2t}e^{4}\,u(t-2)\right] = e^{4}(-2)e^{-2t}u(t-2) + e^{-2t}e^{4}\delta(t-2)$

$= -2e^{4}e^{-2t}u(t-2) + e^{-2(2)}e^{4}\delta(t-2)$

$= -2e^{4}e^{-2t}u(t-2) + \delta(t-2)$

(d) $\int_{-\infty}^{t} \cos(100\pi\tau)[\delta(\tau) + \delta(\tau-.002)]\,d\tau$

$= \int_{-\infty}^{t} \cos(100\pi \cdot 0)\delta(\tau)\,d\tau + \int_{-\infty}^{t} \cos(100\pi(.002))\,\delta(\tau-.002)\,d\tau$

$= 1\,u(t) + \cos(0.2\pi)u(t-.002) = u(t) + 0.809\,u(t-.002)$

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PROBLEM 9.4:

(a) $\left[e^{-(t-4)}u(t-4)\right]\delta(t-5)$

Property: $x(t)\delta(t-5) = x(5)\delta(t-5)$

$$\Rightarrow \left[e^{-(5-4)}u(5-4)\right]\delta(t-5) = e^{-1}\cancelto{1}{u(1)}\,\delta(t-5) = \frac{1}{e}\delta(t-5)$$

(b) $\displaystyle\int_{-\infty}^{t-5}\delta(\tau-1)\,d\tau = \int_{-\infty}^{t-6}\delta(\lambda)\,d\lambda$ $\qquad \lambda = \tau-1$, $d\lambda = d\tau$

$$= u(t-6) - u(-\infty)$$

$$= u(t-6)$$

(c) $\dfrac{d}{dt}\left\{e^{-(t-4)}u(t-4)\right\}$

$$= \left[\frac{d}{dt}e^{-(t-4)}\right]u(t-4) + e^{-(t-4)}\frac{d}{dt}u(t-4)$$

$$= -e^{-(t-4)}u(t-4) + \underbrace{e^{-(t-4)}\delta(t-4)}_{\text{eval at}}\ t=4 \qquad e^{-(t-4)}\Big|_{t=4} = 1$$

$$= -e^{-(t-4)}u(t-4) + \delta(t-4)$$

(d) use shifting property: $\delta(t-a)*\delta(t-b) = \delta(t-(a+b))$

$$\left[\underbrace{\delta(t-1)*\delta(t-2)}\right]*\delta(t)$$

$$\delta(t-3)*\delta(t) = \delta(t-3)$$

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PROBLEM 9.5:

Solve for $h(t)$ in

$$\left[e^{-(t-4)}u(t-4)\right] * h(t) = 2e^{-t}u(t)$$

In order to find $h(t)$, use the shifting property of the impulse:

$$x(t) * \delta(t-t_1) = x(t-t_1)$$

Thus we can write the first term above as

$$e^{-t}u(t) * \delta(t-4) = e^{-(t-4)}u(t-4)$$

Then we must solve:

$$e^{-t}u(t) * \left(\delta(t-4) * h(t)\right) = 2e^{-t}u(t)$$

which requires that

$$\delta(t-4) * h(t) = 2\delta(t)$$

Since $\delta(t-a) * \delta(t-b) = \delta(t-a-b)$ we conclude that $\boxed{h(t) = 2\delta(t+4)}$

i.e., $\delta(t-4) * 2\delta(t+4) = 2\delta(t)$

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PROBLEM 9.6:

(a) $x(t)[\delta(t+1) + \delta(t-1)]$

$= x(t)\delta(t+1) + x(t)\delta(t-1)$ impulse at $t=1$

$= x(-1)\delta(t+1) + x(1)\delta(t-1)$

(b) $\int_{-\infty}^{\infty} x(\tau)\delta(\tau-1)\,d\tau$ impulse at $\tau=1$

$= \int_{-\infty}^{\infty} x(1)\delta(\tau-1)\,d\tau = x(1)\int_{-\infty}^{\infty}\delta(\tau-1)\,d\tau = x(1)$

(c) $\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$ impulse at $\tau=t$

∴ replace τ with t in $x(\tau)$

$= \int_{-\infty}^{\infty} x(t)\delta(t-\tau)\,d\tau$

AREA of $\delta(\;)$ is one

$= x(t)\int_{-\infty}^{\infty}\delta(t-\tau)\,d\tau = x(t)$

(d) $\delta^{(1)}(t) * x(t-1) = x^{(1)}(t-1)$ Using eq. (9.47)

Recall that $x^{(1)}(t)$ is the first derivative.

Thus, $x^{(1)}(t-1) = \frac{d}{dt}x(t-1)$

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PROBLEM 9.7:

(a) $$\left[e^{-3t} + \sin(t)\right]\cdot\left[\delta(t) + \delta(t-1)\right]$$

$$= \underbrace{e^{-3t}\delta(t)}_{\text{eval at } t=0} + e^{-3t}\delta(t-1) + \sin(t)\delta(t) + \underbrace{\sin(t)\delta(t-1)}_{\text{eval at } t=1}$$

$$= e^{-3(0)}\delta(t) + e^{-3(1)}\delta(t-1) + \cancelto{0}{\sin(0)}\,\delta(t) + \sin(1)\delta(t-1)$$

$$= \delta(t) + e^{-3}\delta(t-1) + \sin(1)\delta(t-1)$$

(b) $$\int_{-\infty}^{\infty} \underbrace{\sin(2\pi\tau)\delta(\tau-1)}\, d\tau$$

evaluate integrand at $\tau=1$, before doing the integral

$$= \int_{-\infty}^{\infty} \sin(2\pi\cdot 1)\delta(\tau-1)\, d\tau$$

$\sin(2\pi)$ is zero!

$$= \int_{-\infty}^{\infty} 0\cdot\delta(\tau-1)\, d\tau = 0$$

(c) $$\int_{-\infty}^{\infty} \sin(2\pi\tau)\,\delta(t-\tau)\, d\tau$$

evaluate at $\tau=t$

$$= \int_{-\infty}^{\infty} \sin(2\pi t)\,\delta(t-\tau)\, d\tau$$

Area is 1

$$= \sin(2\pi t)\int_{-\infty}^{\infty} \delta(t-\tau)\, d\tau$$

$$= \sin(2\pi t)$$

Note: the integral is equivalent to a convolution

$$\sin(2\pi t) * \delta(t) = \int_{-\infty}^{\infty} \sin(2\pi\tau)\,\delta(t-\tau)\, d\tau$$

PROBLEM 9.8:

$$y(t) = e^{-at}u(t) * e^{-at}u(t)$$

$$= \int_{-\infty}^{\infty} e^{-a\tau}u(\tau)\, e^{-a(t-\tau)}u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-a\tau} e^{-at} e^{a\tau}\, d\tau \qquad \underline{\underline{\text{If}}}\ t \geq 0$$

$$= \int_{0}^{t} e^{-at}\, d\tau = e^{-at}\int_{0}^{t} d\tau = t e^{-at}$$

If $t<0$, then $u(t)u(t-\tau)=0$ and the integrand is zero. So we can write the final answer as:

$$y(t) = t e^{-at}u(t)$$

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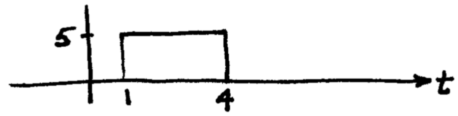
PROBLEM 9.9:

$$h(t) = 5u(t-1) - 5u(t-4)$$



(a)

$$y(t) = u(t) * h(t)$$
$$= u(t) * [5u(t-1) - 5u(t-4)]$$
$$= 5u(t) * u(t-1) - 5u(t) * u(t-4)$$

Use the fact that $u(t) * u(t) = t\,u(t)$ which can be combined with the shift property to write $u(t) * u(t-a) = (t-a)u(t-a)$

Thus, $y(t) = 5(t-1)u(t-1) - 5(t-4)u(t-4)$

(b) The three regions are: $t<1$, $1 \le t \le 4$, and $t>4$.

When $t<1$, both unit-step signals are zero, so $y(t)=0$

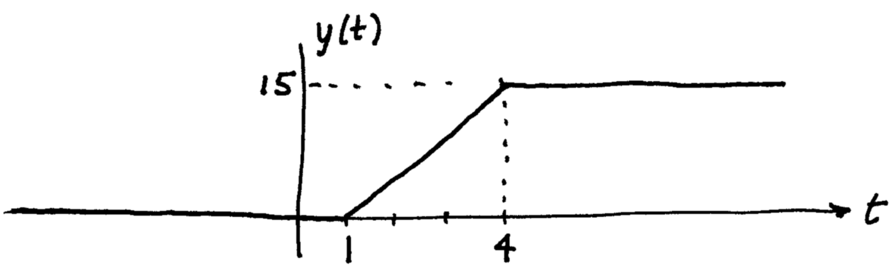
For $1 \le t \le 4$, $u(t-1)=1$ and $u(t-4)=0$, so $y(t)=5t-5$

For $t>4$, $u(t-1)=1$ and $u(t-4)=1$ so $y(t) = 5t-5-5t+20 = 15$

In summary,

$$y(t) = \begin{cases} 0 & t<1 \\ 5t-5 & 1 \le t \le 4 \\ 15 & 4<t \end{cases}$$

(c)



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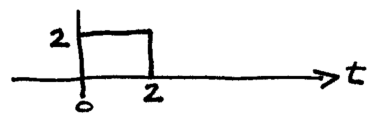
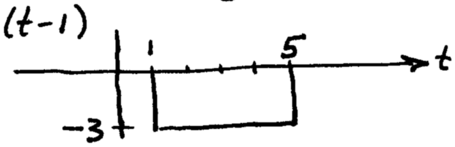
PROBLEM 9.10:

$x(t) = 2u(t) - 2u(t-2)$



$h(t) = 3u(t-5) - 3u(t-1)$



(a)

$y(t) = [2u(t) - 2u(t-2)] * [3u(t-5) - 3u(t-1)]$

Use the fact that $u(t-a) * u(t-b) = (t-(a+b))u(t-(a+b))$

$y(t) = 6u(t) * u(t-5) - 6u(t-2) * u(t-5) - 6u(t) * u(t-1) + 6u(t-2) * u(t-1)$

$= 6(t-5)u(t-5) - 6(t-7)u(t-7) - 6(t-1)u(t-1) + 6(t-3)u(t-3)$

(b) Since the unit steps start at $t = 1, 3, 5$ and 7, the regions are: $t<1$, $1 \le t < 3$, $3 \le t < 5$, $5 \le t < 7$, $7 \le t$.

For $t<1$, all terms are zero, so $y(t) = 0$

For $1 \le t < 3$, only $u(t-1)$ is nonzero, so $y(t) = -6t + 6$

For $3 \le t < 5$, $u(t-1)$ & $u(t-3)$ are nonzero, so $y(t) = -6t + 6 + 6t - 18$
$= -12$

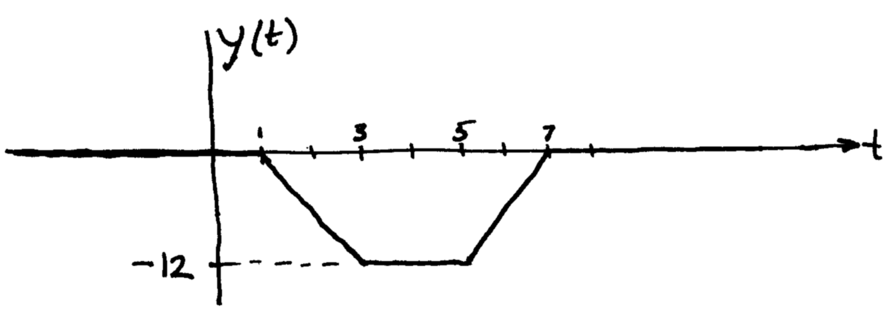
For $5 \le t < 7$, $y(t) = -6t + 6 + 6t - 18 + 6t - 30 = 6t - 42$

For $7 \le t$, $y(t) = -6t + 6 + 6t - 18 + 6t - 30 - 6t + 42 = 0$

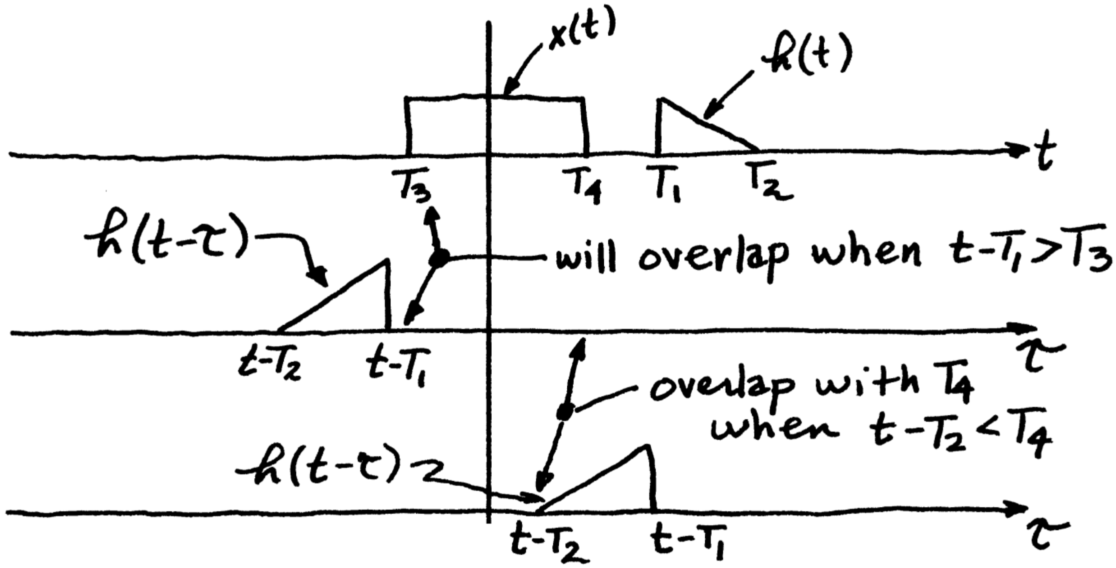
In summary,

$$y(t) = \begin{cases} 0 & t < 1 \\ -6t + 6 & 1 \le t < 3 \\ -12 & 3 \le t < 5 \\ 6t - 42 & 5 \le t < 7 \\ 0 & 7 \le t \end{cases}$$

(c)



PROBLEM 9.11:



Thus, there is overlap of $h(t-\tau)$ & $x(\tau)$ when $t-T_1 > T_3$ and $t-T_2 < T_4$

$$\Rightarrow \quad T_1 + T_3 < t < T_2 + T_4$$

So the end points of $y(t)$ are

$$T_5 = T_1 + T_3 \quad \text{and} \quad T_6 = T_2 + T_4$$

PROBLEM 9.12:

$$h(t) = e^{-0.1(t-2)}\left(u(t-2) - u(t-12)\right)$$

(a) The system is stable because $\int |h(t)|dt < \infty$

$$\int_{-\infty}^{\infty} |h(t)|dt = \int_{2}^{12} |e^{-0.1(t-2)}|dt < \int_{2}^{12} dt = 10 < \infty$$

(b) The system is causal because $h(t)=0$ for $t<0$.

A plot of $h(t)$ starts at $t=2$.

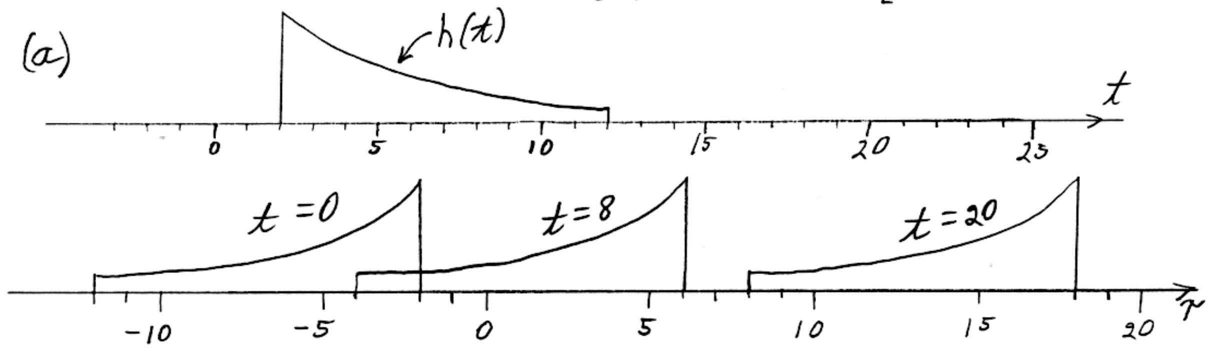
(c) $x(t) = \delta(t-2)$

$$\Rightarrow \quad y(t) = \delta(t-2) * h(t)$$
$$= h(t-2)$$
$$= e^{-0.1(t-4)}\left(u(t-4) - u(t-14)\right)$$

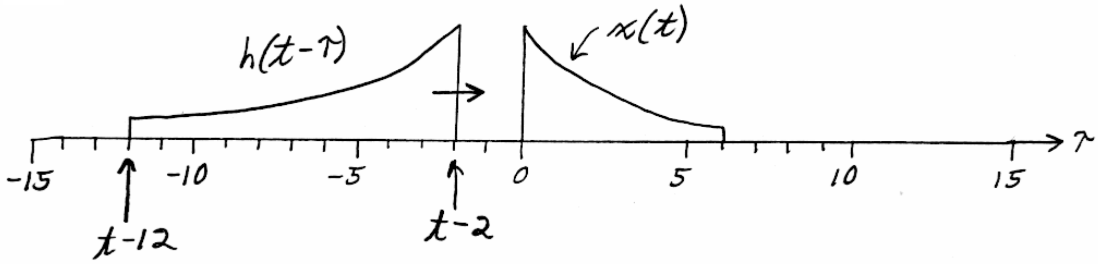
PROBLEM 9.13:

$$h(t) = e^{-0.1(t-2)}\left[u(t-2) - u(t-12)\right]$$

(a)



(b) $x(t) = e^{-0.25t}\left[u(t) - u(t-6)\right]$



Region 1 $\quad t-2<0, \therefore t<2 \qquad y(t)=0$

Region 2 $\quad 0<t-2<6, \therefore 2<t<8 \qquad \int_0^{t-2}$

Region 3 $\quad t-2>6 \qquad t-12<0$

$\therefore 8<t<12 \qquad \int_0^{6}$

Region 4 $\quad 0<t-12<6$

$\therefore 12<t<18 \qquad \int_{t-12}^{6}$

Region 5 $\quad t-12>6 \quad \therefore t>18 \qquad y(t)=0$

In each case, the integrand is

$$x(t)\,h(t-\tau) = e^{-0.1(t-2)}\,e^{-0.15\tau}$$

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PROBLEM 9.13 (more):

Region 1 $\boxed{y(t) = 0 \quad \text{for } t < 2}$

Region 2 $y(t) = e^{-0.1(t-2)} \int_0^{t-2} e^{-0.15\tau} d\tau =$

$$= \frac{e^{-0.1(t-2)}}{-0.15} \left[e^{-0.15\tau} \right|_0^{t-2} = \boxed{6.667 \left[e^{-0.1(t-2)} - e^{-0.25(t-2)} \right] \quad \text{for } 2 < t < 8}$$

Region 3 $y(t) = e^{-0.1(t-2)} \int_0^{6} e^{-0.15\tau} d\tau$

$$= \frac{e^{-0.1(t-2)}}{-0.15} \left[e^{-0.15\tau} \right|_0^{6} = \frac{e^{-0.1(t-2)}}{0.15} \left[e^{0} - e^{-0.15(6)} \right]$$

$$= \boxed{3.956 e^{-0.1(t-2)} \quad \text{for } 8 < t < 12}$$

Region 4

$$y(t) = e^{-0.1(t-2)} \int_{t-12}^{6} e^{-0.15\tau} d\tau = \frac{e^{-0.1(t-2)}}{-0.15} \left[e^{-0.15\tau} \right|_{t-12}^{6}$$

$$= 6.667 e^{-0.1(t-2)} \left[e^{-0.15(t-12)} - e^{-0.15(6)} \right]$$

$$= \boxed{49.2628 e^{-0.25t} - 2.7108 e^{-0.1(t-2)} \quad \text{for } 12 < t < 18}$$

Region 5

$\boxed{y(t) = 0 \quad \text{for } t > 18}$

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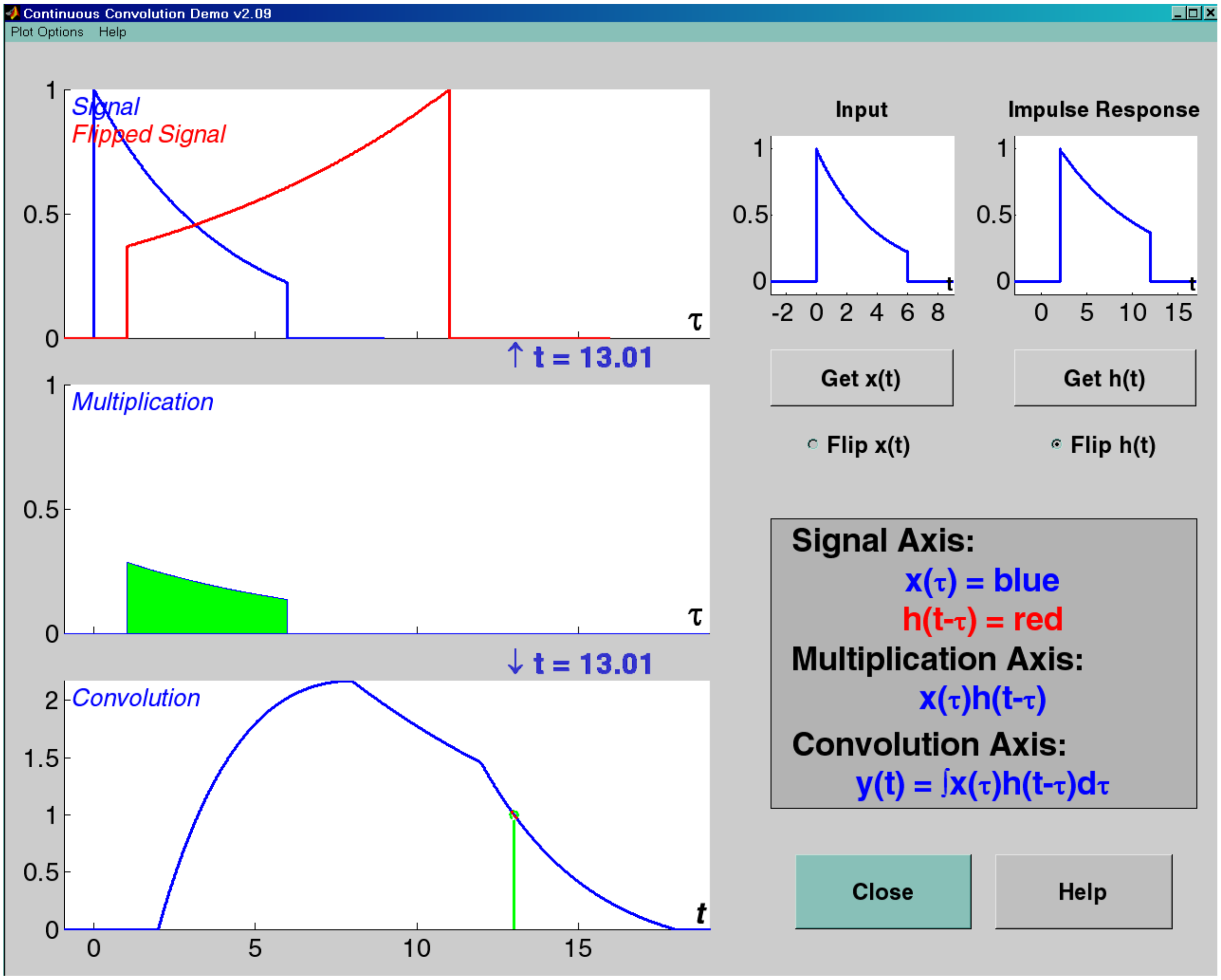
PROBLEM 9.13 (more):

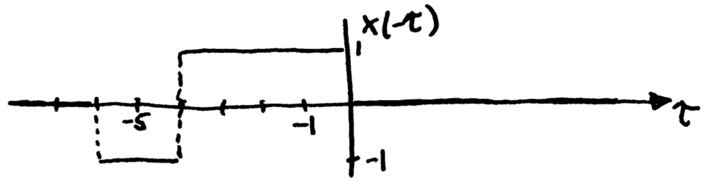
(c) and (d)



PROBLEM 9.14:

(a) $y(0) = \int_{-\infty}^{\infty} h(\tau)\, x(-\tau)\, d\tau$

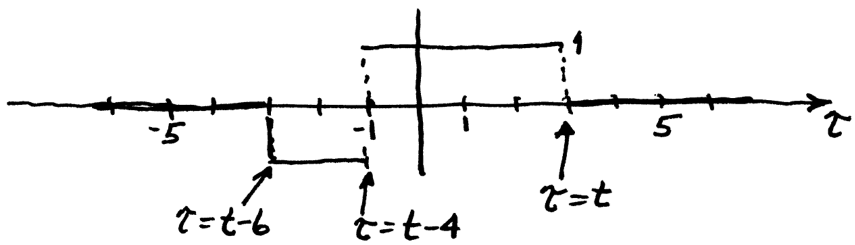
flipped version of $x(t)$



Thus $x(-\tau)$ overlaps $h(\tau)$ from -1 to 0.

$$y(0) = \int_{-1}^{0} (\tau+1)\, d\tau = \text{area of triangle} = \frac{1}{2}$$

(b) As $x(t-\tau)$ is moved by changing t, we can have zero output if (1) there is no overlap of $h(\tau)\, x(t-\tau)$ or (2) if the area from the integral is zero.



No overlap when: $\boxed{t < -1}$ or $t-6 > 1 \Rightarrow \boxed{t > 7}$

The integral gives zero area if the entire triangle for $h(\tau)$ is multiplied by $+1$ or by -1.

$h(\tau)$ times $+1$: when $t \geq 1$ and $t-4 \leq -1$

$$\Rightarrow \quad \boxed{1 \leq t \leq 3}$$

$h(\tau)$ times -1: when $t-4 \geq 1$ and $t-6 \leq -1$

$$\Rightarrow \quad \boxed{t = 5}$$

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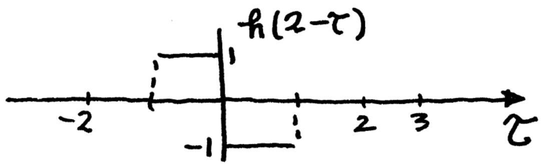
PROBLEM 9.15:

(a) The LTI system is stable because $\int |h(t)|dt < \infty$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_1^3 1\,dt = 2 < \infty$$

(b)



$h(2-\tau)$ is a flip and shift by 2.

(c) Use part (b) for $h(2-\tau)$; $x(\tau) = u(\tau)$

$$y(2) = \int_{-\infty}^{\infty} u(\tau)\, h(2-\tau)\, d\tau$$

$$= \int_0^{\infty} h(2-\tau)\, d\tau$$

$$= \int_0^1 (-1)\, d\tau = -1$$

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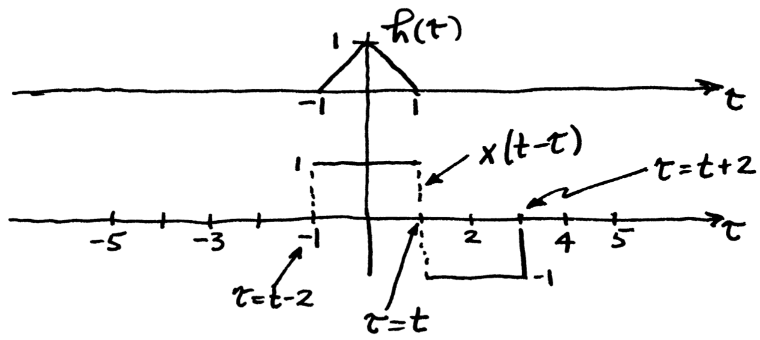
PROBLEM 9.16:

(a) $y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$



No overlap when $t+2 < -1$ or $t-2 > 1$

$\Longrightarrow \boxed{t < -3}$ or $\boxed{t > 3}$

The integral can give zero area if half of the triangle is multiplied by +1 and the other half by −1. This can happen when $\boxed{t = 0}$

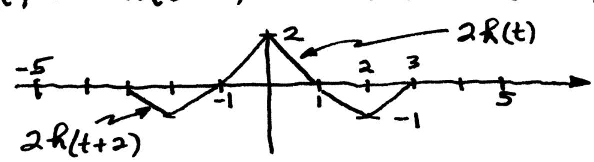
(b) The largest positive output value happens when the triangle is multiplied by +1. This is true when $t=1$ (as drawn above).

$$y(1) = \int_{-1}^{1} h(\tau)\,d\tau = \text{area of triangle} = 1$$

(c) $\frac{d}{dt}y(t)$ can be calculated by taking $\frac{d}{dt}x(t)$ and then doing the convolution

$$\frac{d}{dt}x(t) = -\delta(t+2) + 2\delta(t) - \delta(t-2)$$

$$\frac{d}{dt}y(t) = -h(t+2) + 2h(t) - h(t-2)$$



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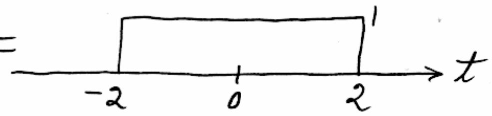
PROBLEM 9.17:

(a) $y(t) = \int_{t-2}^{t+2} x(\tau)\, d\tau$

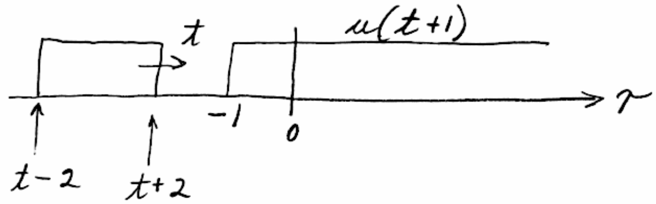
let $x(\tau) = \delta(\tau)$ $\quad h(t) = \int_{t-2}^{t+2} \delta(\tau)\, d\tau = u(\tau)\Big|_{t-2}^{t+2}$

$= u(t+2) - u(t-2) =$ 

(b) Yes, it is stable because $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

(c) No, it is not causal because $h(t) = 1$ for $-2 < t < 0$

(d) $y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} u(t+1)\left[u(t-\tau+2) - u(t-\tau-2)\right] d\tau$



Region 1 $\quad t+2 < -1 \quad t < -3 \quad y(t) = 0$

Region 2 $\quad t+2 \geq -1$ and $t-2 < -1 \qquad 1 > t \geq -3$

$y(t) = \int_{-1}^{t+2} 1 \cdot d\tau = \tau\Big|_{-1}^{t-2} = t+2+1 = t+3$

Region 3 $\quad t-2 > -1 \quad t > 1 \quad y(t) = \int_{t-2}^{t+2} 1 \cdot d\tau = 4$

PROBLEM 9.18:

$$h(t) = \delta(t) - 3e^{-3t}u(t)$$

$$x(t) = u(-t)$$

$$y(t) = x(t) * h(t) = u(-t) * \left[\delta(t) - 3e^{-3t}u(t)\right]$$

$$= u(-t) - 3\underbrace{u(-t) * e^{-3t}u(t)}$$

$$\rightarrow \int_{-\infty}^{\infty} u(-\tau)\, e^{-3(t-\tau)} \underbrace{u(t-\tau)}\, d\tau$$

zero for $\tau > 0$ zero for $\tau > t$

There are two cases: $t \le 0$ and $t > 0$

If $t \le 0$, then the integral is

$$\int_{-\infty}^{t} e^{-3(t-\tau)}\, d\tau = e^{-3t}\int_{-\infty}^{t} e^{3\tau}\, d\tau$$

$$= e^{-3t}\, \frac{e^{3\tau}}{3}\Bigg|_{-\infty}^{t} = \frac{e^{-3t}}{3}\left(e^{3t} - 0\right) = \frac{1}{3}$$

If $t > 0$, then

$$\int_{-\infty}^{0} e^{-3(t-\tau)}\, d\tau = e^{-3t}\int_{-\infty}^{0} e^{3\tau}\, d\tau$$

$$= e^{-3t}\, \frac{e^{3\tau}}{3}\Bigg|_{-\infty}^{0} = \frac{e^{-3t}}{3}(1-0) = \frac{e^{-3t}}{3}$$

Putting these two cases together with unit-steps, we can write: $\frac{1}{3}u(-t) + \frac{1}{3}e^{-3t}u(t)$

Back to $y(t)$:

$$y(t) = u(-t) - 3\left[\frac{1}{3}u(-t) + \frac{1}{3}e^{-3t}u(t)\right]$$

$$= u(-t) - u(-t) - e^{-3t}u(t)$$

$$= -e^{-3t}u(t)$$

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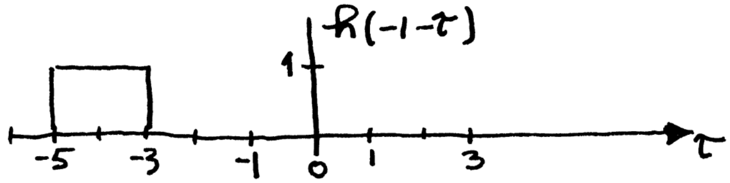
PROBLEM 9.19:

$$h(t) = u(t-2) - u(t-4)$$

(a)



$$h(-1-\tau) = u(-1-\tau-2) - u(-1-\tau-4)$$

$$= u(-\tau-3) - u(-\tau-5) = \begin{cases} 1 & -5 < t < -3 \\ 0 & \text{elsewhere} \end{cases}$$

(b) $$y(t) = x(t) * h(t) = [u(t) - \delta(t-2)] * h(t)$$

$$= u(t) * h(t) - h(t-2)$$

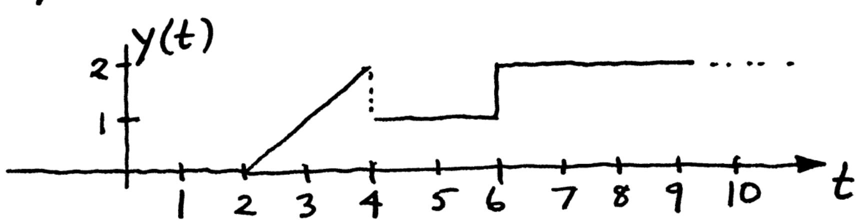
$$u(t) * h(t) = u(t) * [u(t-2) - u(t-4)]$$

$$= u(t) * u(t-2) - u(t) * u(t-4)$$

$$= (t-2)u(t-2) - (t-4)u(t-4)$$

Finally,

$$y(t) = (t-2)u(t-2) - (t-4)u(t-4) - u(t-4) + u(t-6)$$



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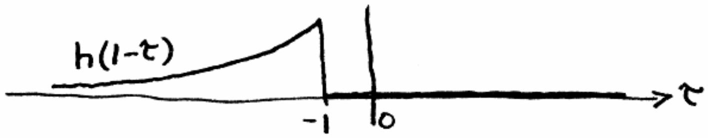
PROBLEM 9.20:

(a) $h(t-\tau)$ for $t=1$ is $h(1-\tau) = e^{-(1-\tau-2)}u(1-\tau-2)$ ← FLIP & SHIFT by 1

$h(1-\tau) = e^{-(-1-\tau)}u(-1-\tau)$ ← STARTS @ $\tau=-1$



(b) Yes, the system is causal because $h(t)=0$ for $t<0$. In fact, $h(t)=0$ for $t<2$.

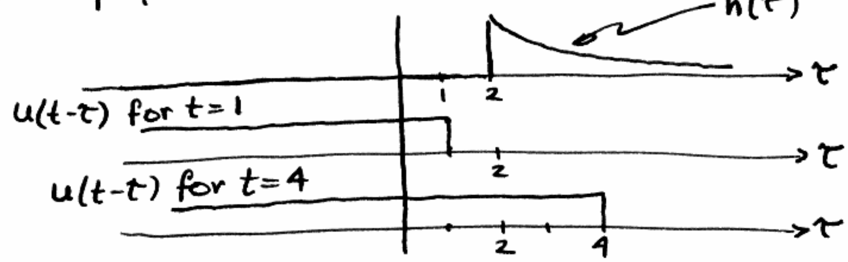
(c) To test for stability we do the integral $\int_{-\infty}^{\infty}|h(t)|dt$

$$\int_{-\infty}^{\infty}|e^{-(t-2)}u(t-2)|dt = \int_{2}^{\infty}e^{-(t-2)}dt = \left.\frac{e^{-(t-2)}}{-1}\right|_{2}^{\infty} = 0 - \frac{e^{0}}{-1} = 1 < \infty$$

Thus the system is stable.

(d) See the result from the convolution below: $t_1 = 2$

(e) Flip & Slide $x(t) = u(t)$



From the drawings, there is NO overlap when $t<2$.

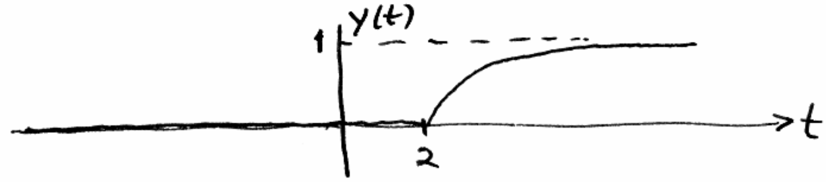
$\Rightarrow y(t) = 0$ for $t<2$.

For $t \geq 2$, we have overlap from $\tau=2$ up to $\tau=t$.

$$y(t) = \int_{2}^{t} 1\cdot e^{-(\tau-2)}d\tau = \left.\frac{e^{-(\tau-2)}}{-1}\right|_{2}^{t}$$

$$y(t) = \frac{e^{-(t-2)}}{-1} - \frac{e^{0}}{-1} = 1 - e^{-(t-2)}$$

$$\therefore y(t) = (1-e^{-(t-2)})u(t-2)$$



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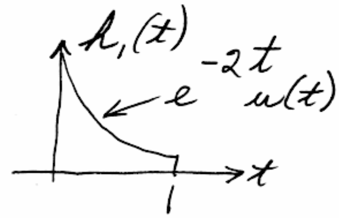
PROBLEM 9.21:

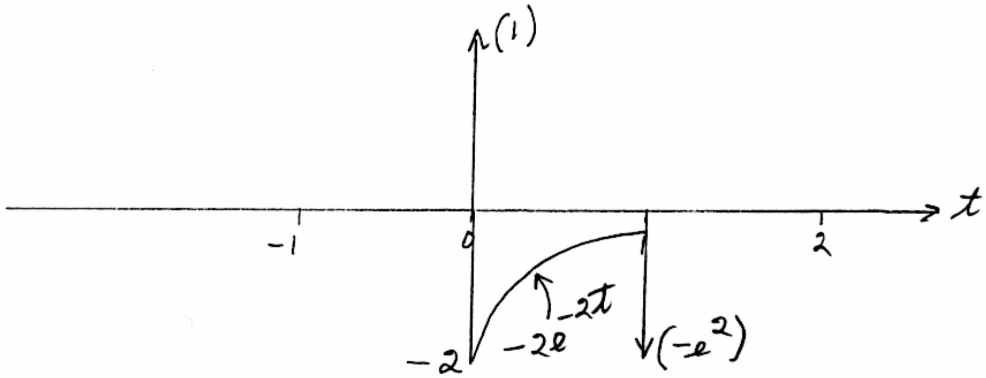
$$y(t) = x(t) * h_1(t) * h_2(t)$$



$$h_1(t) = e^{-2t}\left(u(t) - u(t-1)\right)$$

$$y(t) = \frac{d\,h_1(t)}{dt} = -2e^{-2t}\left(u(t) - u(t-1)\right) + e^{-2t}\left(\delta(t) - \delta(t-1)\right)$$

$$= -2e^{-2t}\left[u(t) - u(t-1)\right] + \quad \delta(t) - e^{2}\delta(t-1)$$

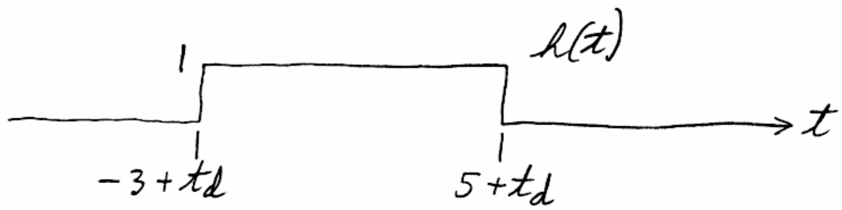


PROBLEM 9.22:

(a) $v(t) = u(t+3) - u(t-5)$

$$h(t) = y(t)\Big|_{x(t)=\delta(t)} = v(t) * \delta(t-t_d) = \delta(t-t_d) * v(t)$$

$$\therefore \; h(t) = u(t+3-t_d) - u(t-5-t_d)$$



(b) $t_d \geq 3$ because $h(t) = 0$ for $t < 0$

(c) #1 and #2 are not stable because

$$\int_{-\infty}^{\infty} |h_1(t)|\,dt \to \infty \quad \text{and} \quad \int_{-\infty}^{\infty} |h_2(t)|\,dt \to \infty$$

#3 is stable

The overall system is stable because

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

PROBLEM 9.23:

First system: $w(t) = x(t) - x(t-2)$

If the input is an impulse, then the output is $\delta(t) - \delta(t-2)$. Then this output is used as the input to the second system whose impulse response is $u(t)$. Thus,

$$y(t) = [\delta(t) - \delta(t-2)] * u(t)$$

$$= u(t) - u(t-2) = \begin{cases} 1 & 0 \le t < 2 \\ 0 & \text{elsewhere} \end{cases}$$

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PROBLEM 9.24:

(a) when $x(t) = \delta(t)$

$$w_1(t) = \delta(t+1) \quad \text{and} \quad w_2(t) = \delta(t-2)$$

$$\Rightarrow v(t) = w_1(t) - w_2(t) = \delta(t+1) - \delta(t-2)$$

The impulse response of an integrator is $u(t)$

$$\Rightarrow y(t) = u(t) * [\delta(t+1) - \delta(t-2)]$$

$$= u(t+1) - u(t-2) \quad \text{This is } h(t)$$

$$h(t) = u(t+1) - u(t-2) = \begin{cases} 1 & -1 \le t < 2 \\ 0 & \text{elsewhere} \end{cases}$$

(b) The overall system is <u>NOT</u> causal

Because $h(t) \neq 0$ for $t < 0$

(c) The overall system is stable

Because $\displaystyle\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-1}^{2} 1\,dt = 3 < \infty$

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PROBLEM 9.25:

(a) when $x(t) = \delta(t)$

$$\Rightarrow w(t) = e^{-3t}u(t)$$

$$\Rightarrow y(t) = \frac{d}{dt}w(t) = e^{-3t}\delta(t) - 3e^{-3t}u(t)$$

$$= \delta(t) - 3e^{-3t}u(t)$$

$$\therefore\ h(t) = \delta(t) - 3e^{-3t}u(t)$$

(b) when $x(t) = u(t)$, use convolution to get $y(t)$

$$y(t) = x(t) * h(t)$$

$$= u(t) * \left[\delta(t) - 3e^{-3t}u(t)\right]$$

$$= u(t) - 3\left(\frac{1}{3-0}\right)\left(u(t) - e^{-3t}u(t)\right)$$

$$= u(t) - u(t) + e^{-3t}u(t) = e^{-3t}u(t)$$

Easier way: Flip the order—differentiate first

$$u(t) \longrightarrow \boxed{d/dt} \xrightarrow{\delta(t)} \boxed{e^{-3t}u(t)} \longrightarrow y(t) = e^{-3t}u(t)$$

$\delta(t)$, because $\frac{d}{dt}u(t) = \delta(t)$

(c) when $x(t) = u(t) - u(t-10)$, use LTI

$$u(t) \longrightarrow e^{-3t}u(t)$$

$$u(t-10) \longrightarrow e^{-3(t-10)}u(t-10) \quad \text{(Time-Invariance)}$$

$$\Rightarrow y(t) = e^{-3t}u(t) - e^{-3(t-10)}u(t-10)$$