

$$\text{Let } f(x) = (x-2)^2 - \ln x$$

$$f'(x) = 2(x-2) - \frac{1}{x}$$

Newton's method

$$p_n = p_{n-1} - \frac{(p_{n-1}-2)^2 - \ln p_{n-1}}{2(p_{n-1}-2) - \frac{1}{p_{n-1}}}$$

With  $p_0 = 1$ , compute  $p_1$

$$p_1 = 1 - \frac{(1-2)^2 - \ln 1}{2(1-2) - \frac{1}{1}} = 1 - \frac{1}{-2-1} = 1 + \frac{1}{3} = \frac{4}{3}$$

Secant method

$$p_n = p_{n-1} - \frac{[(p_{n-1}-2)^2 - \ln p_{n-1}](p_{n-1} - p_{n-2})}{(p_{n-1}-2)^2 - \ln p_{n-1} - (p_{n-2}-2)^2 + \ln p_{n-2}}$$

With  $p_0 = 1$  and  $p_1 = 2$  compute  $p_2$

$$p_2 = 2 - \frac{[-\ln 2](2-1)}{-\ln 2 - (1-2)^2 + \ln 1} = 2 - \frac{\ln 2}{\ln 2 + 1}$$
$$= \frac{\ln 2 + 2}{\ln 2 + 1}$$