

1.3 Algorithms and Convergence

1) Convergence.

Def: Suppose $\{\beta_n\}_{n=1}^{\infty}$ is a sequence which converges to zero, and $\{d_n\}_{n=1}^{\infty}$ converges to d . If there is a constant k

$$|d_n - d| \leq k |\beta_n| \quad \text{for large } n$$

we say that $\{d_n\}_{n=1}^{\infty}$ converges to d with rate of convergence $O(\beta_n)$.

In most cases we use

$$\beta_n = \frac{1}{n^p} \quad \text{for some } p > 0.$$

② Ex. $d_n = \frac{n+3}{n^2+1} \quad d_n \rightarrow 0$

$$d_n \leq \frac{n+n}{n^2} = \frac{2n}{n^2} = 2 \cdot \frac{1}{n} \quad \forall n \geq 3$$

$\Rightarrow d_n \rightarrow 0$ with rate of convergence $O(\frac{1}{n})$,

③ Ex: $d_n = \frac{5}{n^5+5} \leq \frac{5}{n^5} \Rightarrow d_n \rightarrow 0$ with $O(\frac{1}{n^5})$