## PROBLEM 2.1:

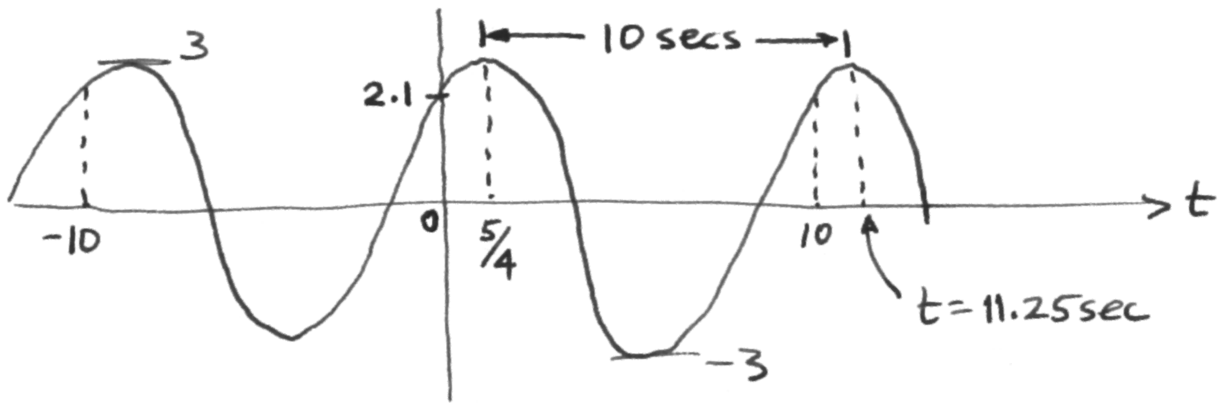
$$x(t) = 3\cos\left(\frac{\pi}{5}t - \frac{\pi}{4}\right)$$

$$2\pi f = \frac{\pi}{5} \implies f = \frac{1}{10} \implies T = 10 \text{ sec. (PERIOD)}$$

$$\varphi = -2\pi\frac{t_1}{T} \implies t_1 = -\frac{\varphi}{2\pi}T = \frac{\pi/4}{2\pi}\times 10 = \frac{5}{4} = 1.25 \text{sec}$$

at $t=0$

$$x(t) = 3\cos(-\pi/4) = \frac{3\sqrt{2}}{2} \approx 2.1$$

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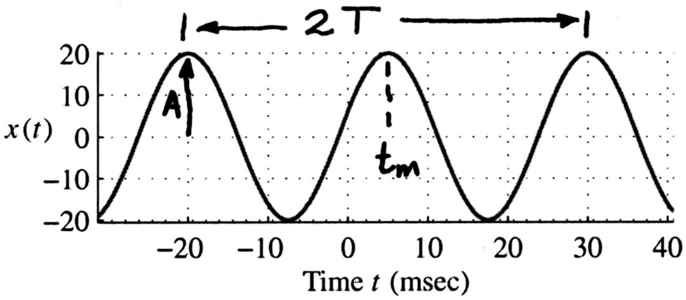
## PROBLEM 2.2:




Period: $2T = (30-(-20))\text{ msec} = 50\text{ msec}$

$\Rightarrow T = 25\text{ msec}$

Frequency: $\omega_0 = 2\pi/T = 2\pi\left(\frac{1}{25\times10^{-3}}\right) = 2\pi(40)\text{ rad/s}$

$f = 40\text{ Hz}$

Amplitude: $A = 20$

Time-Shift: $t_m = +5\text{ msec}$

Phase: $\varphi = -\omega_0 t_m = -2\pi(40)\times5\times10^{-3}$

$\varphi = -2\pi(0.2) = -0.4\pi$

$x(t) = 20\cos(80\pi t - 0.4\pi)$

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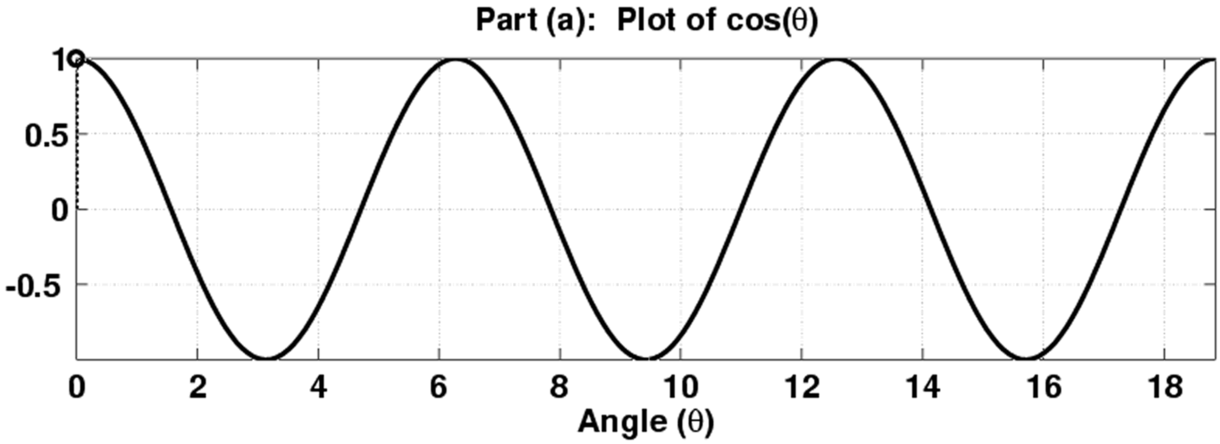
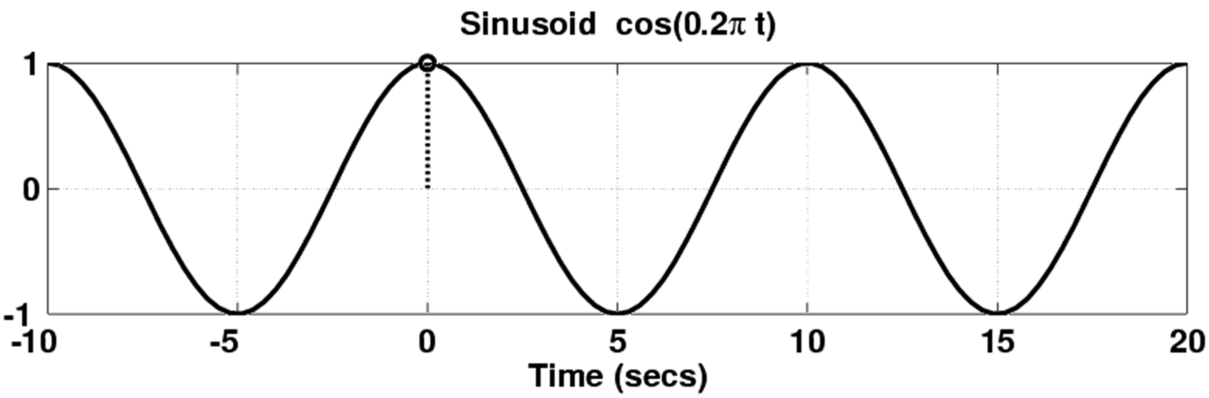
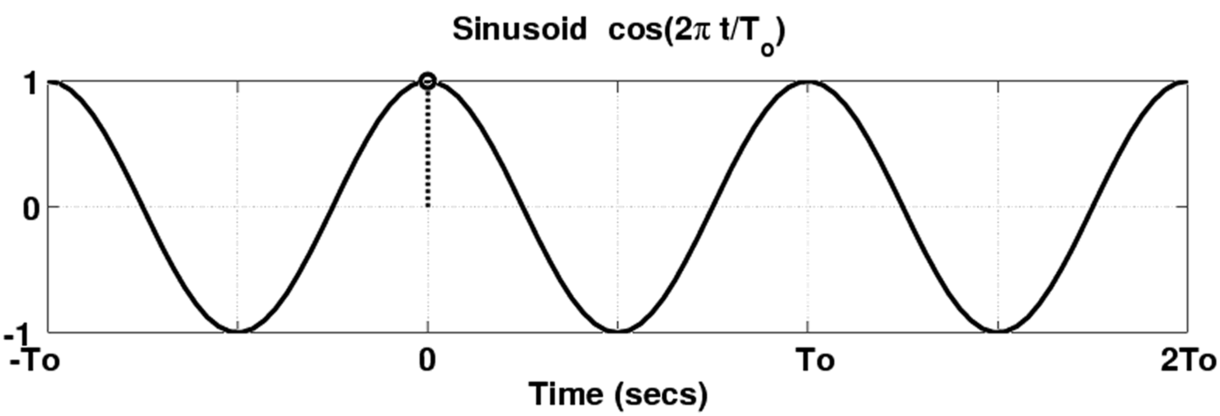
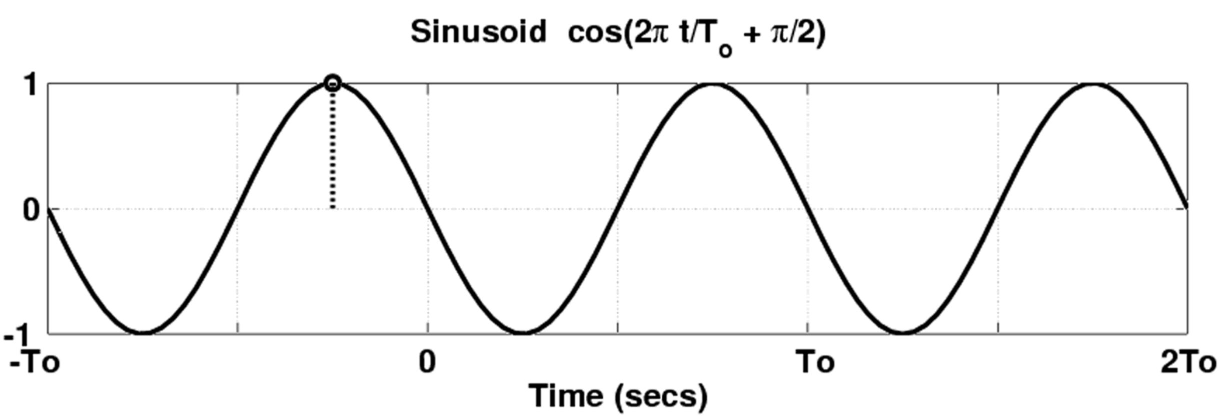
## PROBLEM 2.3:

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## PROBLEM 2.4:

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \cdots$$

$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} + \cdots$$

Separate the real and imaginary parts:

$$e^{j\theta} = \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right)}_{\cos\theta} + j\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)}_{\sin\theta}$$

$$\therefore\ e^{j\theta} = \cos\theta + j\sin\theta$$

which proves Euler's formula.

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## PROBLEM 2.5:

(a)
$$\cos(\theta_1+\theta_2) = \Re e\{e^{j(\theta_1+\theta_2)}\} = \Re e\{e^{j\theta_1}e^{j\theta_2}\}$$
$$= \Re e\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)\}$$
$$= \Re e\{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 + j(\text{other terms})\}$$
$$= \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$

(b)
$$\cos(\theta_1-\theta_2) = \Re e\{e^{j(\theta_1-\theta_2)}\} = \Re e\{e^{j\theta_1}e^{-j\theta_2}\}$$
$$= \Re e\{(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 - j\sin\theta_2)\}$$
$$= \Re e\{\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 + j(\text{other terms})\}$$
$$= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$$

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## PROBLEM 2.6:

$$(\cos\theta + j\sin\theta)^n = (e^{j\theta})^n = e^{jn\theta}$$

$$e^{jn\theta} = \cos(n\theta) + j\sin(n\theta)$$

Thus,

$$(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)$$

$$\left(\frac{3}{5} + j\frac{4}{5}\right)^{100} = \left(e^{j0.927}\right)^{100} = \left(e^{j0.295167\pi}\right)^{100}$$

$$= e^{j29.5167\pi}$$

$$= e^{j1.5167\pi}$$

$$\boxed{\text{BECAUSE } e^{j28\pi} = 1}$$

$$= \cos(1.5167\pi) + j\sin(1.5167\pi)$$

$$= \cos(273^\circ) + j\sin(273^\circ)$$

$$= 0.0525 - j0.9986$$

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## PROBLEM 2.7:

(a) $3e^{j\pi/3} + 4e^{-j\pi/6} = \left(\frac{3}{2} + j\frac{3\sqrt{3}}{2}\right) + \left(4\frac{\sqrt{3}}{2} - j\frac{4}{2}\right)$

$= 4.9641 + j0.5981$

$= 5e^{j0.12}$ NOTE: $0.12$ rad $= 6.87°$

---

(b) $\sqrt{3} - j3 = \sqrt{3+3^2}\, e^{-j\pi/3} = \sqrt{12}\, e^{-j\pi/3}$.

$\Rightarrow \left(\sqrt{3} - j3\right)^{10} = \left(\sqrt{12}\, e^{-j\pi/3}\right)^{10}$

$= 2^{10}\, 3^5\, e^{-j\,10\pi/3}$ $\qquad -\frac{10\pi}{3} + 4\pi = \frac{-10\pi + 12\pi}{3} = 2\pi/3$

$= 248{,}832\, e^{+j2\pi/3} = -124{,}416 + j215{,}494.83$

---

(c) $\frac{1}{\sqrt{3} - j3} = \frac{1}{\sqrt{12}\, e^{-j\pi/3}} = \frac{1}{\sqrt{12}} e^{+j\pi/3} = 0.2887 e^{+j\pi/3}$

$= 0.14434 + j0.25$

---

(d) $\left(\sqrt{3} - j3\right)^{1/3} = \left(\sqrt{12}\, e^{-j\pi/3}\right)^{1/3} = \left(\sqrt{12}\, e^{-j(\pi/3 + 2\pi\ell)}\right)^{1/3}$

$= 12^{1/6}\, e^{-j(\pi/9 + \frac{2\pi}{3}\ell)}$ $\qquad \ell =$ integer

Need $\ell = 0, 1, 2$

There are 3 answers:

$1.513\, e^{-j\pi/9} = 1.422 - j0.5175$

$1.513\, e^{-j7\pi/9} = -1.159 - j0.9726$

$1.513\, e^{-j13\pi/9} = 1.513\, e^{+j5\pi/9} = -0.2627 + j1.49$

---

(e) $\Re e\{j e^{-j\pi/3}\} = \Re e\{e^{j\pi/2} e^{-j\pi/3}\}$.

$= \Re e\{e^{j\pi/6}\} = \cos(\pi/6) = \frac{\sqrt{3}}{2} = 0.866$

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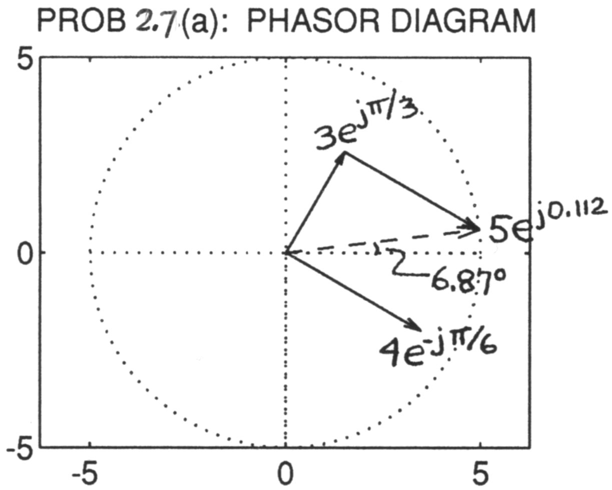
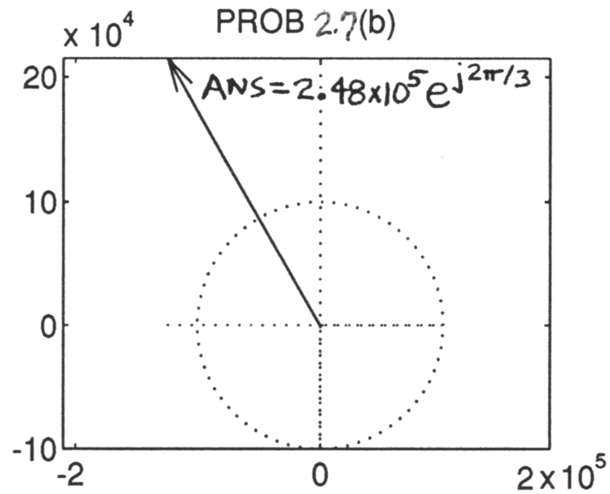
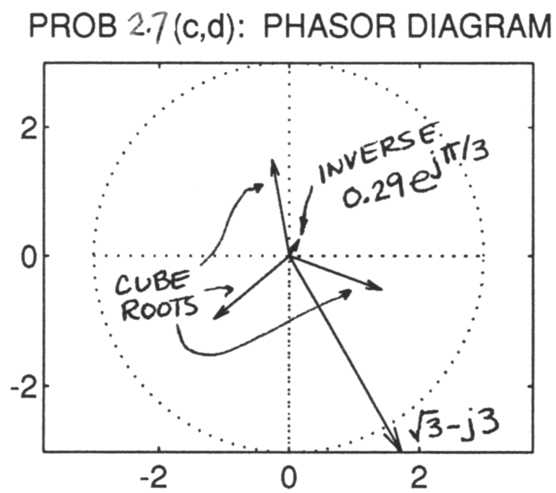
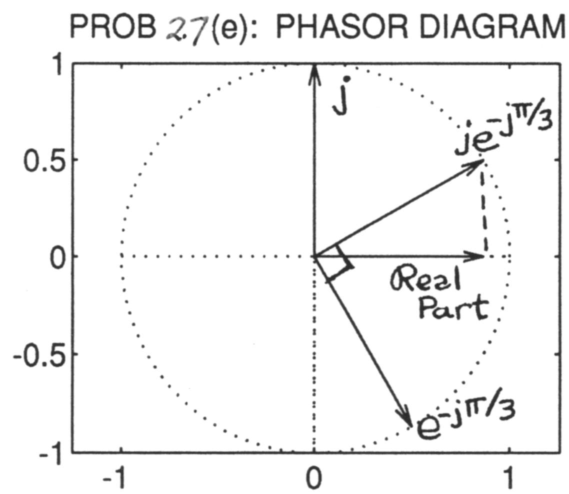
## PROBLEM 2.7 (more):

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## PROBLEM 2.8:

```
J = sqrt(-1);
dt = 1/100;
tt = -1 : dt : 1;
Fo = 2;
xx = 300*real( exp( J*(2*pi*Fo*(tt - 0.75) ) ) );
%
subplot(2,1,1)
plot( tt, xx ),   grid
title( 'SECTION of a SINUSOID' ), xlabel('TIME   (sec)')
```

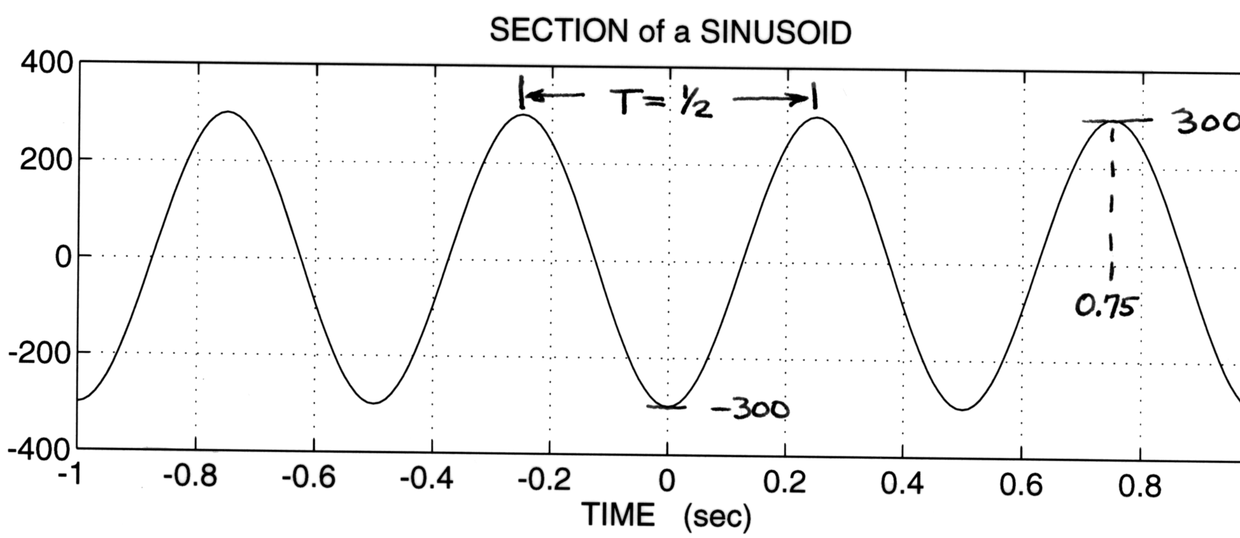
$$x(t) = 300\,\mathcal{Re}\left\{ e^{j2\pi F_0(t-0.75)} \right\}$$

$$\boxed{F_0 = 2} \Rightarrow T = \tfrac{1}{2}\text{ sec}$$

$$= 300\cos\left(4\pi(t - 3/4)\right)$$

@ $t=0$ $\quad x(0) = 300\cos(-3\pi) = -300$

@ $t=3/4$ $\quad x(3/4) = 300\cos(0) = 300$ ← POSITIVE PEAK

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## PROBLEM 2.9:

(a) Use phasors to perform the addition

$$2\sin(\omega_0 t + \pi/4) = 2\cos(\omega_0 t - \pi/4) \rightarrow 2e^{-j\pi/4}$$

$$\cos(\omega_0 t) \longrightarrow 1e^{j0}$$

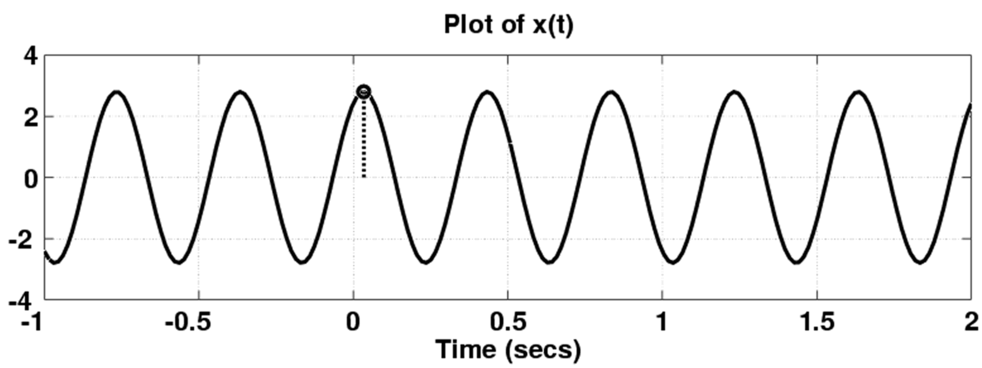
$$Ae^{j\varphi} = 2e^{-j\pi/4} + 1 = \left(\tfrac{2}{\sqrt{2}} + 1\right) - j\tfrac{2}{\sqrt{2}} = 2.798e^{-j0.17\pi}$$

$$x(t) = 2.798\cos(\omega_0 t - 0.17\pi)$$

ANGLE $= -30.36^\circ$ or $-0.53$ rads

(b) $\omega_0 = 5\pi \Rightarrow$ period is $T = \frac{2\pi}{\omega_0} = \frac{2}{5}$

Therefore, the interval $[-1, 2]$ contains $\frac{3}{(2/5)} = 7\frac{1}{2}$ periods




(c) We already found the magnitude & phase in part (a), so

$$z(t) = 2.798e^{-j0.17\pi}e^{j\omega_0 t}$$

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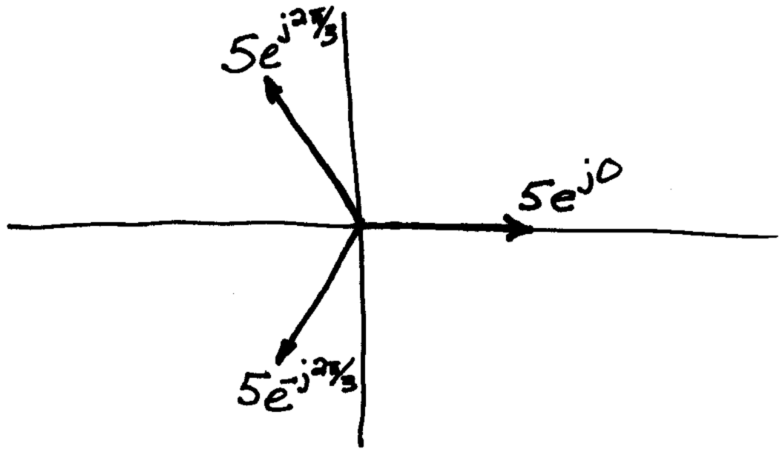
## PROBLEM 2.10:

Use Phasors:

$$5\cos(\omega t) \longrightarrow 5e^{j0} = 5 + j0$$

$$5\cos(\omega t + 120^\circ) \longrightarrow 5e^{j2\pi/3} = -\frac{5}{2} + j5\frac{\sqrt{3}}{2}$$

$$5\cos(\omega t - 120^\circ) \longrightarrow 5e^{-j2\pi/3} = -5/2 - j5\frac{\sqrt{3}}{2}$$




Vector Sum:

$$5 + \left(-\frac{5}{2} + j5\frac{\sqrt{3}}{2}\right) + \left(-\frac{5}{2} - j5\frac{\sqrt{3}}{2}\right)$$

$$= \left(5 - \frac{5}{2} - \frac{5}{2}\right) + j\left(5\frac{\sqrt{3}}{2} - 5\frac{\sqrt{3}}{2}\right) = 0$$

Thus, $x(t) = 0$

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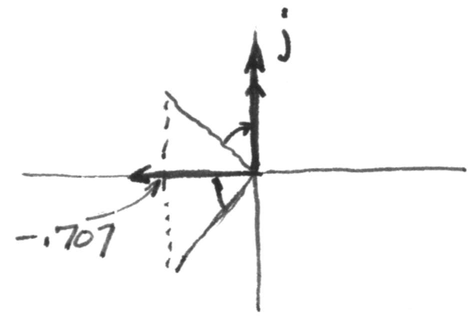
## PROBLEM 2.11:

$$\mathcal{R}e\{(1+j)e^{j\theta}\} = -1$$

$$\Rightarrow \mathcal{R}e\{\sqrt{2}\, e^{j\pi/4}\, e^{j\theta}\} = -1$$

$$\Rightarrow \mathcal{R}e\{e^{j(\theta+\pi/4)}\} = -1/\sqrt{2}$$




$$\Rightarrow \cos(\theta+\pi/4) = -1/\sqrt{2}$$

$$\therefore \quad \underbrace{\theta+\pi/4}_{} = \begin{cases} 3\pi/4 \\ -3\pi/4 \end{cases}$$

ROTATE BY $\pi/4$

$$\Rightarrow \theta = \pi/2 \text{ or } -\pi$$

i.e. $e^{j\theta} = -1$ or $j$

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## PROBLEM 2.12:

$$\frac{d^2 x(t)}{dt^2} = -100\, x(t)$$

Try $x(t) = e^{j\omega t}$ and solve for $\omega$.

$$\frac{dx(t)}{dt} = j\omega e^{j\omega t} \longrightarrow \frac{d^2 e^{j\omega t}}{dt^2} = -\omega^2 e^{j\omega t}$$

Plug into differential equation:

$$-\omega^2 e^{j\omega t} = -100\, e^{j\omega t}$$

$$\Rightarrow \omega^2 = 100 \Rightarrow \omega = \pm 10$$

$$x(t) = e^{j10t} \quad \text{or} \quad x(t) = e^{-j10t}$$

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## PROBLEM 2.13:

(a) $s_i(t) = \Im m\{5e^{j\pi/3}e^{j10\pi t}\} = \Im m\{5e^{j(10\pi t + \pi/3)}\}$

$\Rightarrow s_i(t) = 5\sin(10\pi t + \pi/3)$

We can convert to the cosine form:

$s_i(t) = 5\cos(10\pi t - \pi/6)$  $\sin\theta = \cos(\theta - \pi/2)$

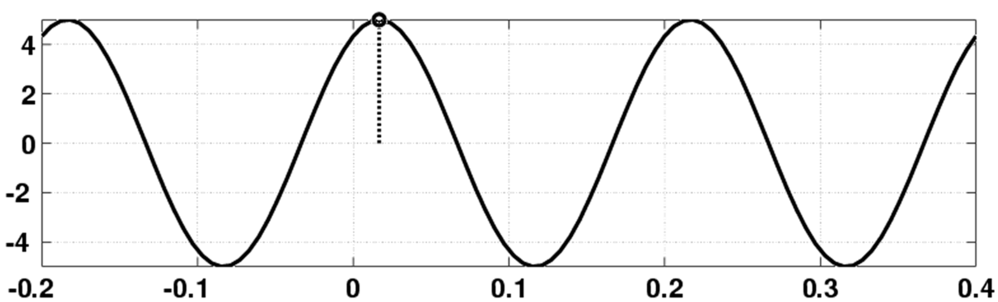
The period of $s_i(t)$ is $T = 1/5$, because $\frac{2\pi}{T} = 10\pi$

The value at $t=0$ is $s_i(0) = 5\cos(-\pi/6) = \frac{5\sqrt{3}}{2} = 4.33$

The peak is at $t = t_1$ where

$10\pi t_1 - \pi/6 = 0 \Rightarrow t_1 = 1/60$

Plot of $s_i(t)$

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## PROBLEM 2.13 (more):

(b) $q(t) = \Im m\{\dot{s}(t)\} = \Im m\{(5e^{j\pi/3})(j10\pi e^{j10\pi t})\}$

$$= \Im m\{50\pi e^{j(10\pi t + \pi/3 + \pi/2)}\}$$

$$= 50\pi \sin(10\pi t + 5\pi/6)$$

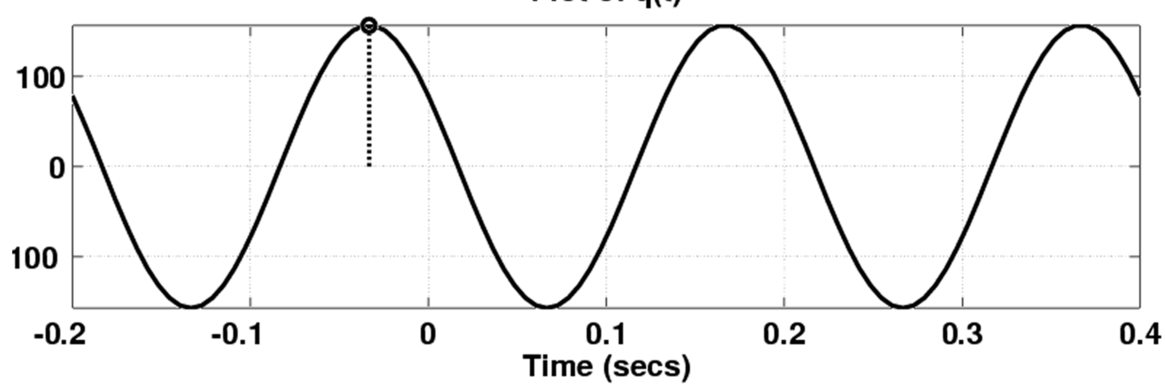
$$= 50\pi \cos(10\pi t + \pi/3)$$

The period of $q(t)$ is also $T = 1/5$.

$q(0) = 50\pi \cos(\pi/3) = 25\pi = 78.54$

The max value of $q(t)$ is at $t_1$ which solves: $10\pi t_1 + \pi/3 = 0 \Rightarrow t_1 = -1/30$

Plot of q(t)

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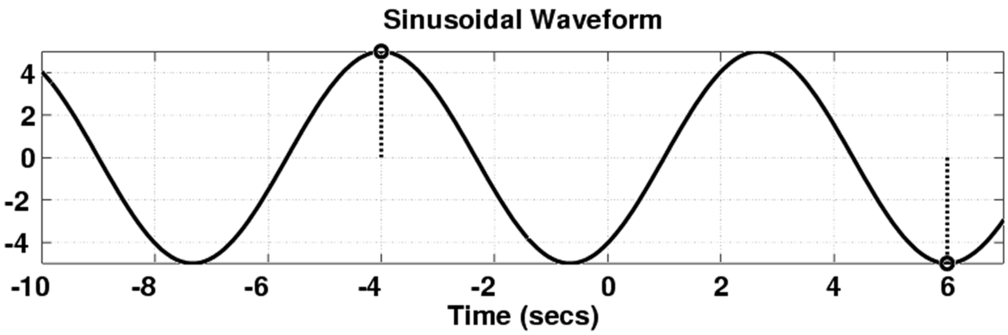
# PROBLEM 2.14:

From the graph we can get the following information:

positive peak at $t = -4$ msec, value $= 5$

negative peak at $t = 6$ msec




There are $1\frac{1}{2}$ periods from $t=-4$ms to $t=6$ms.

$$(1\tfrac{1}{2})T = 10 \text{ msec}$$

$$\Rightarrow T = \frac{20}{3} \text{ msec} = 6\tfrac{2}{3} \text{ msec}$$

$$\omega_0 = 2\pi/T = 2\pi/(20/3000) = 300\pi \text{ rad/sec}$$

Phase: $$\varphi = -2\pi\left(\frac{t_1}{T}\right) = -2\pi\left(\frac{-4}{20/3}\right) = \frac{12\pi}{10} = 1.2\pi$$

$$\therefore\ x(t) = 5\cos(300\pi t + 1.2\pi)$$

For the complex notation $X = \text{Mag}\, e^{j\text{phase}}$

$$X = 5e^{j1.2\pi}$$

$$x(t) = \Re e\left\{5e^{j1.2\pi} e^{j300\pi t}\right\}$$

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## PROBLEM 2.15:

Express $x(t) = 5\cos(\omega t + \frac{1}{3}\pi) + 7\cos(\omega t - \frac{5}{4}\pi) + 3\cos(\omega t)$ in the form $x(t) = A\cos(\omega t + \phi)$.

*Solution:*
Convert to phasors:

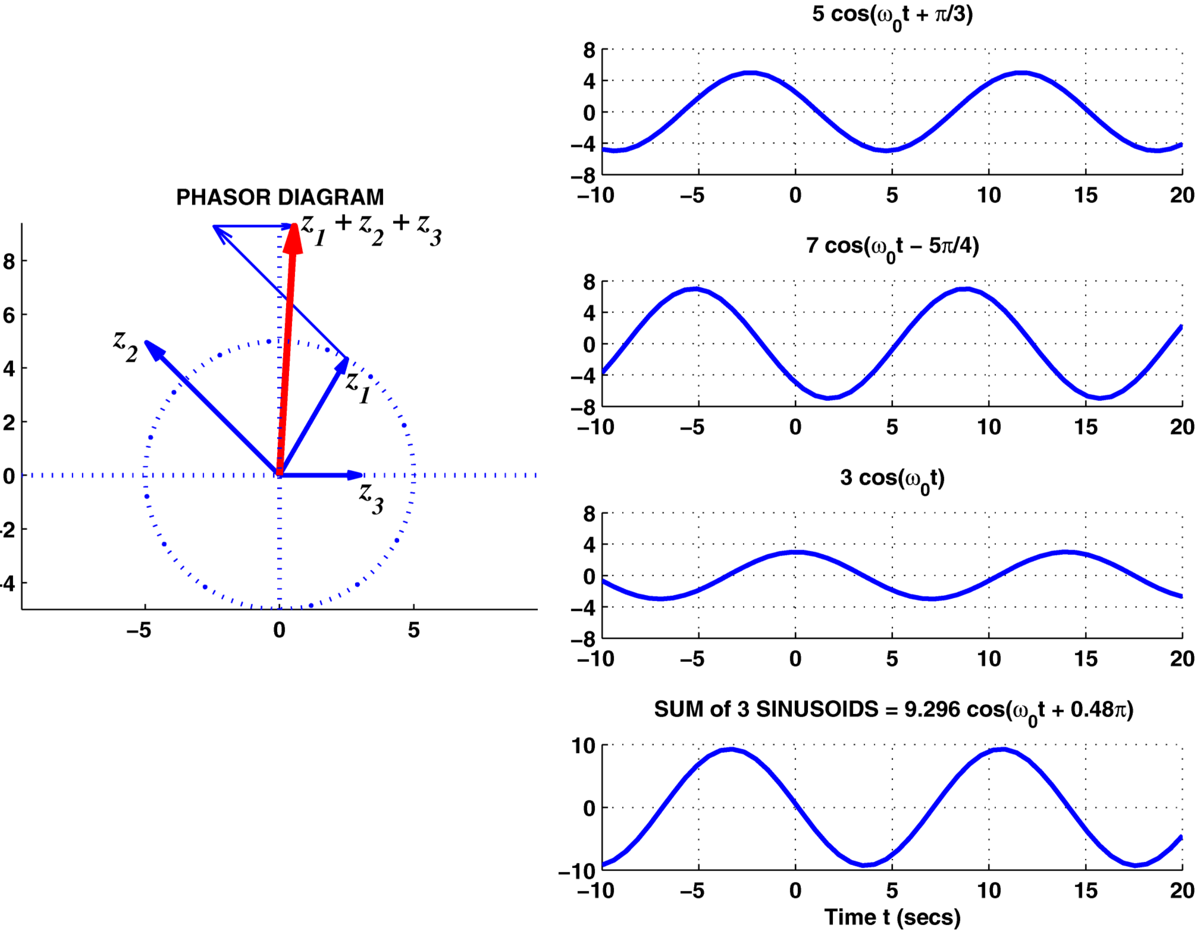
$$5\cos(\omega t + \tfrac{1}{3}\pi) \longrightarrow z_1 = 5e^{j\pi/3} = 2.5 + j4.33$$
$$7\cos(\omega t - \tfrac{5}{4}\pi) \longrightarrow z_2 = 7e^{j5\pi/34} = -4.95 + j4.95$$
$$3\cos(\omega t) \longrightarrow z_3 = 3e^{j0} = 3 + j0$$

Perform the phasor addition to get:

$$z_1 + z_2 + z_3 = (2.5 + j4.33) + (-4.95 + j4.95) + (3) = 0.5503 + j9.28 = 9.296e^{j0.48\pi}$$

Thus, the resultant sinusoid is:

$$x(t) = 9.296\cos(\omega t + 0.48\pi)$$

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## PROBLEM 2.16:

The phase of a sinusoid can be related to time shift:

$$x(t) = A\cos(2\pi f_\circ t + \phi) = A\cos(2\pi f_\circ (t - t_1))$$

In the following parts, assume that the period of the sinusoidal wave is $T = 8$ sec.

(a) "When $t_1 = -2$ sec, the value of the phase is $\phi = \pi/2$."
Explain whether this is TRUE or FALSE.

$$\varphi = -2\pi \frac{t_1}{T} = -2\pi \frac{(-2)}{8} = \frac{4\pi}{8} = \frac{\pi}{2} \quad \therefore \text{TRUE}$$

(b) "When $t_1 = 3$ sec, the value of the phase is $\phi = 3\pi/4$."
Explain whether this is TRUE or FALSE.

$$\varphi = -2\pi \frac{t_1}{T} = -2\pi \frac{3}{8} = -\frac{3\pi}{4} \quad \therefore \text{FALSE}$$

(c) "When $t_1 = 7$ sec, the value of the phase is $\phi = \pi/4$."
Explain whether this is TRUE or FALSE.

$$\varphi = -2\pi \frac{t_1}{T} = -2\pi \frac{7}{8} = -\frac{7\pi}{4} \longrightarrow -\frac{7\pi}{4} + 2\pi = \frac{\pi}{4} \quad \therefore \text{TRUE}$$

BUT you can add multiple of $2\pi$

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## PROBLEM 2.17:

$$x(t) = 5\cos(\omega_0 t + 3\pi/2) + 4\cos(\omega_0 t + 2\pi/3) + 4\cos(\omega_0 t + \pi/3)$$

(a) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$ by finding the numerical values of $A$ and $\phi$.

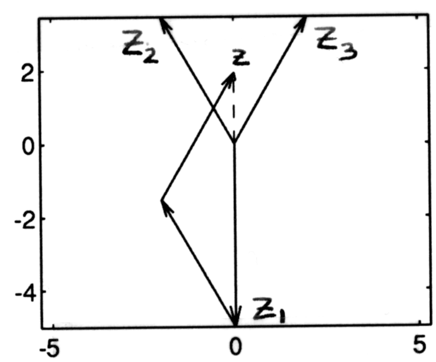
$$\left.\begin{aligned} Z_1 &= 5e^{j3\pi/2} = 0 - 5j \\ Z_2 &= 4e^{j2\pi/3} = -2 + j3.46 \\ Z_3 &= 4e^{j\pi/3} = 2 + j3.46 \end{aligned}\right\} \quad \begin{aligned} Z &= Z_1 + Z_2 + Z_3 \\ &= 0 + j1.928 \\ &= 1.928\, e^{j\pi/2} \end{aligned}$$

$$\therefore \quad x(t) = 1.928\cos(\omega_0 t + \pi/2)$$

***

| Z = X | + jY | Magnitude | Phase | Ph/pi | Ph(deg) |
|---|---|---|---|---|---|
| -9.185e-16 | -5 | 5 | -1.571 | -0.500 | -90.00 |
| -2 | 3.464 | 4 | 2.094 | 0.667 | 120.00 |
| 2 | 3.464 | 4 | 1.047 | 0.333 | 60.00 |
| 4.441e-16 | 1.928 | 1.928 | 1.571 | 0.500 | 90.00 |

(b) Plot all the phasors used to solve the problem in part (a) in the complex plane.

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## PROBLEM 2.18:

Use the fact that $\sin\omega_0 t = \cos(\omega_0 t - \pi/2)$

Then convert to a "phasor equation"

$$\begin{bmatrix} 1 \\ -j \end{bmatrix} = \begin{matrix} 1e^{j0} = A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2} \\ 1e^{-j\pi/2} = 2A_1 e^{j\varphi_1} + A_2 e^{j\varphi_2} \end{matrix} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} A_1 e^{j\varphi_1} \\ A_2 e^{j\varphi_2} \end{bmatrix} \begin{matrix} \leftarrow z_1 \\ \leftarrow z_2 \end{matrix}$$

Solve the simultaneous equations

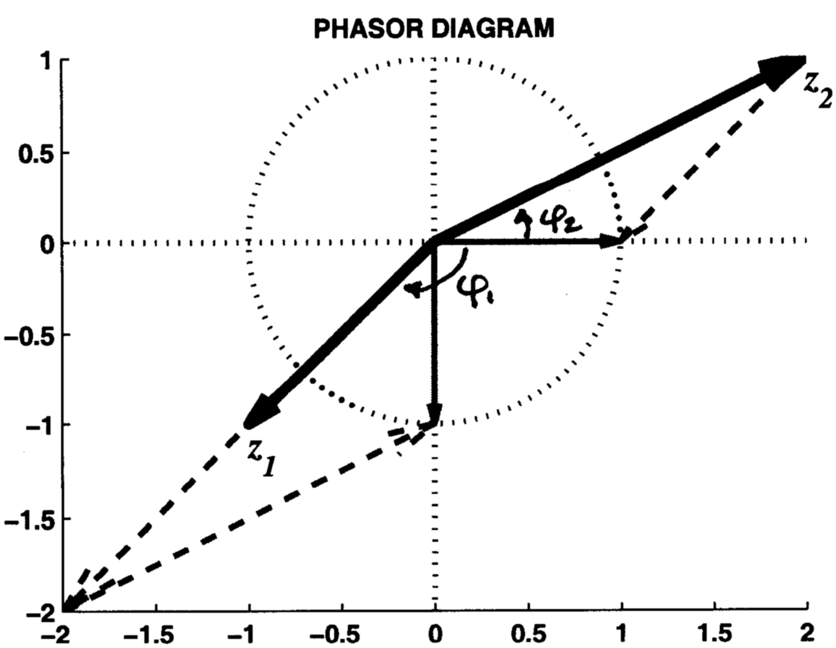
$$\left.\begin{matrix} 1 = z_1 + z_2 \\ -j = 2z_1 + z_2 \end{matrix}\right\} \Rightarrow \begin{matrix} -j - 1 = z_1 \\ 2 + j = z_2 \end{matrix}$$

$$z_1 = A_1 e^{j\varphi_1} = \sqrt{2}\, e^{-j3\pi/4}$$

$$z_2 = A_2 e^{j\varphi_2} = \sqrt{5}\, e^{j0.148\pi}$$

$A_1 = \sqrt{2} = 1.414$ $\qquad$ $A_2 = \sqrt{5} = 2.236$

$\varphi_1 = -3\pi/4$ rads $\qquad$ $\varphi_2 = 0.148\pi$ rads $= 26.57°$



## PROBLEM 2.19:

$$5\cos\omega_0 t = M\cos(\omega_0 t - \pi/6) + 5\cos(\omega_0 t + \psi)$$

phasors: $\downarrow$

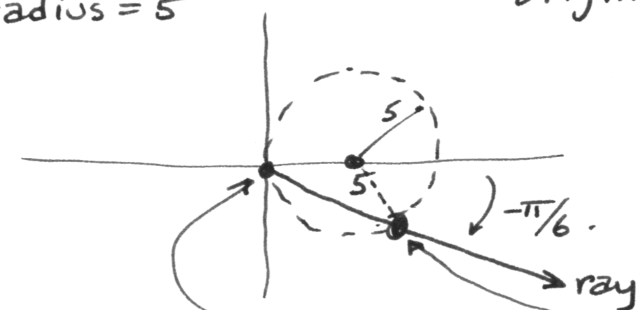
$$5e^{j0} = Me^{-j\pi/6} + 5e^{j\psi}$$

Geometric Approach:

rearrange eqn: $\underbrace{5 - 5e^{j\psi}} = Me^{-j\pi/6}$

As $\psi$ varies 0 to $2\pi$ this side defines a circle with center at 5, radius = 5

As M varies this side defines a ray from the origin at angle $-\pi/6$




∴ One solution is $M=0$ & $\psi=0$

The other is $M=5\sqrt{3}$ & $\psi = 2\pi/3$

ALGEBRAIC APPROACH:

⇒ EQUATE REAL & IMAG PARTS

$$5 = M\cos\pi/6 + 5\cos\psi$$

$$0 = -M\sin\pi/6 + 5\sin\psi \Rightarrow M = 10\sin\psi$$

$$\therefore\ 5 = 5\sqrt{3}\sin\psi + 5\cos\psi = 5\,\Re e\{(1-j\sqrt{3})e^{j\psi}\}.$$

$$\Rightarrow 1 = \Re e\{2e^{-j\pi/3}e^{j\psi}\}.$$

$$\Rightarrow \psi = 0$$

$$\text{or } \psi = 2\pi/3 \Rightarrow M = 10\sin 120^\circ = 5\sqrt{3}$$

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## PROBLEM 2.20:

$$x[n] = 7e^{j(0.22\pi n - 0.25\pi)}$$

$$y[n] = 7e^{j(0.22\pi(n+1) - 0.25\pi)} - 14e^{j(0.22\pi n - 0.25\pi)} + 7e^{j(0.22\pi(n-1) - 0.25\pi)}$$

$$= 7e^{-j0.25\pi} e^{j0.22\pi n}\Big(\underbrace{e^{j0.22\pi} - 2 + e^{-j0.22\pi}}_{2\cos(0.22\pi) - 2 \,=\, -0.459 \,=\, 0.459e^{j\pi}}\Big)$$

$\therefore$

$$y[n] = 7(0.459e^{j\pi})e^{-j0.25\pi}e^{j0.22\pi n}$$

$$= 3.213\, e^{j0.75\pi} e^{j0.22\pi n}$$

$$\boxed{\begin{aligned} \hat{\omega}_0 &= 0.22\pi \\ A &= 3.213 \\ \varphi &= 0.75\pi \end{aligned}}$$

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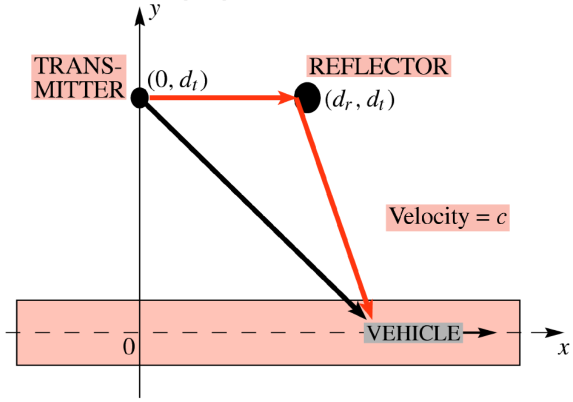
## PROBLEM 2.21:

In a mobile radio system a transmitting tower sends a sinusoidal signal, and a mobile user receives not one but two copies of the transmitted signal: a direct-path transmission and a reflected-path signal (e.g., from a large building) as depicted in the following figure.




The received signal is the sum of the two copies, and since they travel different distances they have different time delays, i.e.,

$$r(t) = s(t - t_1) + s(t - t_2)$$

The distance between the mobile user in the vehicle at $x$ and the transmitting tower is always changing. Suppose that the direct-path distance is

$$d_1 = \sqrt{x^2 + d_t^2} \quad \text{(meters)}$$

where $d_t = 1000$ meters, and where $x$ is the position of the vehicle moving along the $x$-axis. Assume that the reflected-path distance is

$$d_2 = d_r + \sqrt{(x - d_r)^2 + d_t^2} \quad \text{(meters)}$$

where $d_r = 55$ meters.

(a) The amount of the delay (in seconds) can be computed for both propagation paths, by converting distance into time delay by dividing by the speed of light ($c = 3 \times 10^8$ m/s).

$$t_1 = d_1/c = \frac{\sqrt{x^2 + d_t^2}}{c} = \frac{\sqrt{x^2 + 10^6}}{3 \times 10^8} \text{ secs.}$$

$$t_2 = d_2/c = \frac{d_r + \sqrt{(x - d_r)^2 + d_t^2}}{c} = \frac{55 + \sqrt{(x - 55)^2 + 10^6}}{3 \times 10^8} \text{ secs.}$$

(b) When the transmitted signal is $s(t) = \cos(300\pi \times 10^6 t)$, the general formula for the received signal is:

$$r(t) = s(t - t_1) + s(t - t_2) = \cos(300\pi \times 10^6 (t - t_1)) + \cos(300\pi \times 10^6 (t - t_2))$$

When $x = 0$ we can calculate $t_1$ and $t_2$, and then perform a phasor addition to express $r(t)$ as a sinusoid with a known amplitude, phase, and frequency. When $x = 0$, the time delays are

$$t_1 = \frac{\sqrt{0^2 + 10^6}}{3 \times 10^8} = 3.3333 \times 10^{-6} \text{ secs.}$$

$$t_2 = \frac{55 + \sqrt{(0-55)^2 + 10^6}}{3 \times 10^8} = 3.5217 \times 10^{-6} \text{ secs.}$$

Thus we must perform the following addition:

$$\begin{aligned} r(t) &= \cos(300\pi \times 10^6(t - 3.3333 \times 10^{-6})) + \cos(300\pi \times 10^6(t - 3.5217 \times 10^{-6})) \\ &= \cos(300\pi \times 10^6 t - 1000\pi)) + \cos(300\pi \times 10^6 t - 1056.5113579\pi) \end{aligned}$$

As a phasor addition, we carry out the following steps (since $1000\pi$ and $1056\pi$ are integer multiples of $2\pi$):

$$\begin{aligned} R &= 1e^{j0} + 1e^{j0.5113579\pi} \\ &= 1 + j0 + (-0.035674 + j0.99936) \\ &= 0.9643 + j0.9994 = 1.389e^{j0.803} = 1.389e^{j0.256\pi} = 1.389\angle 46.02^\circ \end{aligned}$$

From the polar form of the phasor $R$, we can write $r(t)$ as a sinusoid:

$$r(t) = 1.389 \cos(300\pi \times 10^6 t + 0.256\pi)$$

(c) In order to find the locations where the signal strength is zero, we note that the phase of the two delayed sinusoids must differ by an odd multiple of $\pi$ in order to get cancellation. Thus,

$$\begin{aligned} (2\ell + 1)\pi &= \phi_1 - \phi_2 = -\omega t_1 - (-\omega t_2) \\ &= -300\pi \times 10^6 \left( \frac{\sqrt{x^2 + 10^6}}{3 \times 10^8} - \frac{55 + \sqrt{(x-55)^2 + 10^6}}{3 \times 10^8} \right) \\ &= -\pi \left( \sqrt{x^2 + 10^6} - 55 - \sqrt{(x-55)^2 + 10^6} \right) \end{aligned}$$

The general solution to this equation is difficult, involving a quartic. However, if we choose $\ell = 27$ so that the left hand side becomes $55\pi$, then the $55\pi$ term on the right hand side will cancel, and we obtain an equation in which squaring both sides will produce the answer.

$$\begin{aligned} \pi\sqrt{x^2 + 10^6} &= -\pi\sqrt{(x-55)^2 + 10^6} \\ \Longrightarrow x^2 + 10^6 &= (x-55)^2 + 10^6 \\ \Longrightarrow x^2 &= x^2 - 110x + 55^2 \\ \Longrightarrow 110x &= 55^2 \\ \Longrightarrow x &= \left(\frac{55}{110}\right) 55 = 27.5 \text{ meters} \end{aligned}$$

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## PROBLEM 2.21 (more):

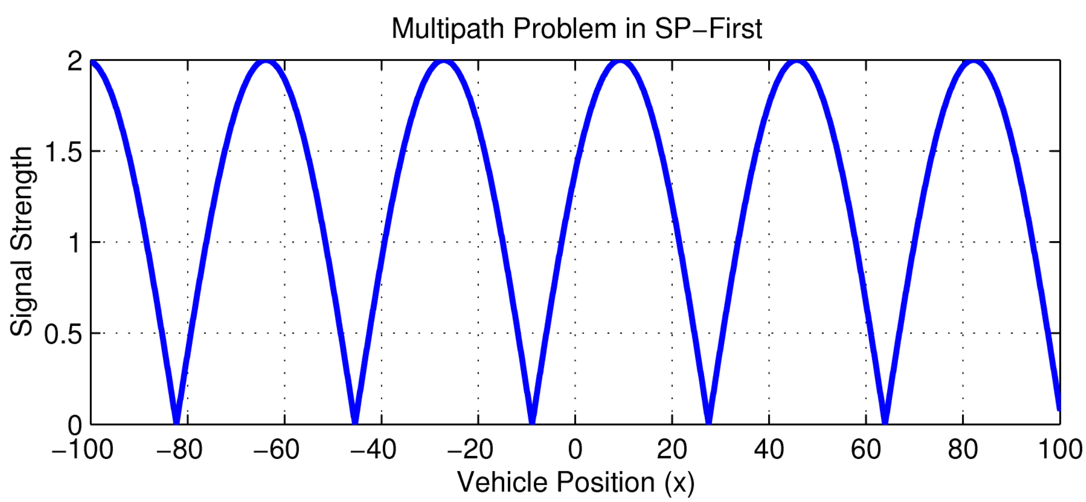
The general solution would be done in the following manner:

$$
\begin{aligned}
-(2\ell+1) &= \sqrt{x^2+10^6} - 55 - \sqrt{(x-55)^2+10^6} \\
\Rightarrow 55-(2\ell+1) &= \sqrt{x^2+10^6} - \sqrt{(x-55)^2+10^6} \\
\Rightarrow 55^2 - 110(2\ell+1) + (2\ell+1)^2 &= x^2 + 10^6 - 2\sqrt{x^2+10^6}\sqrt{(x-55)^2+10^6} + (x-55)^2 + 10^6 \\
\Rightarrow 2\sqrt{x^2+10^6}\sqrt{(x-55)^2+10^6} &= -4\ell^2 + 216\ell + 109 - 55^2 + x^2 + 2\times 10^6 + (x-55)^2
\end{aligned}
$$

Squaring both sides would eliminate the square roots, but would produce a fourth-degree polynomial that would have to be solved for the vehicle position $x$.

(d) Here is a MATLAB script that will plot the signal strength versus vehicle position $x$, thus demonstrating that there are numerous locations where no signal is received (note the null at $x = 27.5$).

```
xx = -100:0.05:100;
d1 = sqrt(xx.*xx + 1e6);
d2 = 55 + sqrt((xx-55).*(xx-55)+1e6);
omeg = 300e6*pi;   c = 3e8;
phi1 = -omeg*d1/c;
phi2 = -omeg*d2/c;
RR = 1*exp(j*phi1) + 1*exp(j*phi2);
subplot('Position',[0.1,0.1,0.6,0.3]);
hp = plot(xx,abs(RR)); grid on,
xlabel('Vehicle Position (x)');
ylabel('Signal Strength');
title('Multipath Problem in SP-First');
set(hp,'LineWidth',2);
print -dpdf multipathResult.pdf
```




Over the range $-100 \le x \le 100$ the nulls appear to be equally spaced 36.4 meters apart, but they are not uniform. A plot over the range $0 \le x \le 1500$ would demonstrate the non-uniformity.