

So $P_n(x)$ can be written as

$$(\star) P_n(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + \dots + f[x_0, x_1, \dots, x_n](x-x_0)\dots(x-x_{n-1})$$

Def: This formula is called Newton's interpolatory (forward) divided-difference formula.

Ex 1: #4/p.131 (a) Construct the interpolating polynomial of degree 4 for the points in the table

x	f(x)	I st DD	II nd DD	III rd DD	IV th DD
0.0	-6.000000				
0.1	-5.89483	1.0517			
0.3	-5.65014	1.22345	0.5725		
0.6	-5.17788	1.5742	0.7015	0.215	
1.0	-4.28172	2.2404	0.9517	0.278	0.06
1.1	-3.99583	2.8589	1.237	0.356625	0.0786

$$P_4(x) = -6 + 1.0517x + 0.5725x(x-0.1) + 0.215x(x-0.1)(x-0.3) + 0.063x(x-0.1)(x-0.3)(x-0.6)$$

(b) Add $f(1.1) = -3.99583$ to the table, and construct the interpolating polynomial of degree 5

The Vth DD is 0.063
 $0.078625 \rightarrow \approx 0.0142$

$$P_5(x) = P_4(x) + 0.0142x(x-0.1)(x-0.3)(x-0.6)(x-1)$$