Examples:

a) $(10011.01)_2 = 2^4 + 2 + 1 + 2^{-2} = (19.25)_{10}$

b) $(.0101\overline{01}\ldots)_2 = 2^{-2} + 2^{-4} + 2^{-6} + \ldots = \sum_{k=1}^{\infty} 2^{-2k} = \sum_{k=1}^{\infty} \left(\frac{1}{4}\right.$

$$= \frac{1}{4}\,\frac{1}{1-\frac{1}{4}} = \frac{1}{4}\cdot\frac{1}{\frac{3}{4}} = \frac{1}{3} = (.33\overline{3}\ldots)_{10}$$

c) $\frac{1}{5} = (0.2)_{10} = .0011\,\overline{0011}\ldots$

To convert we multiply by 2. If the resulting number is larger than 1 we write 1 and subtract one from it, otherwise we write 0 and multiply again by 2.

$$\frac{2}{5} \to \frac{4}{5} \to \frac{8}{5} \;\Big|\; \frac{3}{5} \to \frac{6}{5} \;\Big|\; \frac{1}{5} \;\ldots.$$

Check: $(\overline{0011}\ldots)_2 = 2^{-3} + 2^{-4} + 2^{-7} + 2^{-8} + 2^{-11} + 2^{-12} + \ldots$
$= 2^{-3}(1 + 2^{-4} + 2^{-8} + 2^{-12} + \ldots) + 2^{-4}(1 + 2^{-4} + 2^{-8})$
$= \left(\frac{1}{8} + \frac{1}{16}\right)\cdot\frac{1}{1-\frac{1}{16}} = \frac{3}{16}\cdot\frac{1}{\frac{15}{16}} = \frac{3}{15} = \frac{1}{5}$

3) Machine numbers.

A machine has only finite number digits which can store. The numbers stored in the machines are called <u>floating-point numbers</u>.