

Def: A sequence $p_n \rightarrow p$ of order α if

$$\lim_{n \rightarrow \infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha} = \lambda > 0$$

- if $\alpha = 1$ $0 < \lambda < 1$ - converges linearly
- if $\alpha = 2$ $0 < \lambda$ - converges quadratically

Ex: Determine whether the sequence
converges to zero linearly or quadratically
 $p_n = 10^{-2n}$

Solution: $p_{n+1} = 10^{-2(n+1)}$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{10^{-2(n+1)}}{(10^{-2n})^\alpha} &= \lim_{n \rightarrow \infty} \frac{10^{-2(n+1)}}{10^{-2n\alpha}} = \\ &= \lim_{n \rightarrow \infty} 10^{-2n(1-\alpha)} \cdot 10^{-2} = \underbrace{10^{-2}}_{\text{if } \alpha = 1} \end{aligned}$$

Thus, the sequence converges linearly.

5) Lagrange Interpolation

Given: x_0, \dots, x_n ($n+1$) points
 f_0, \dots, f_n

we can write a polynomial of degree n