

The error in composite trapezoidal rule is

$$E_n^T(f) = -\frac{b-a}{12} h^2 f''(\xi)$$

where $a < \xi < b$.

$$f(x) = x^2 \cos x, \quad a = 0, \quad b = \pi$$

$$f'(x) = 2x \cos x - x^2 \sin x$$

$$f''(x) = 2 \cos x - 2x \sin x - 2x \sin x - x^2 \cos x$$

$$= (2 - x^2) \cos x - 4x \sin x$$

Crude estimate:

$$|f''(x)| \leq (\pi^2 - 2) + 4\pi \leq 20$$

$$\frac{\pi}{12} h^2 \cdot 20 \leq 10^{-4}$$

$$h^2 \leq \frac{12}{\pi \cdot 20 \cdot 10^4} = \frac{6}{\pi \cdot 10 \cdot 10^4} = \frac{3}{5 \cdot 10^4 \pi}$$

$$h \leq \frac{1}{100} \sqrt{\frac{3}{5\pi}}$$

$$\frac{\pi}{n} \leq \frac{1}{100} \sqrt{\frac{3}{5\pi}} \quad n \geq 100\pi \sqrt{\frac{5\pi}{3}}$$