Ex #2/3/131 Use Newton's forward-difference formula to construct the interpolating polynomial of degree 3. Use Newton's backward-difference formula to construct the interpolating polynomial of degree 3. Use either polynomial to approximate $f(-\frac{1}{3})$.

x	$f(x)$	Δf	$\Delta^2 f$	$\Delta^3 f$
-0.75	-0.0718125			
-0.5	-0.02475	0.0470625		
-0.25	0.334937 5	0.3596875	0.312625	
0	1.101	0.7660625	0.40637 5	0.09375

$$P_3(x_0+sh) = P_3(s) = -0.0718125 + 0.0470625s + 0.312625\frac{s(s-1)}{2!} + 0.09375\frac{s(s-1)(s-2)}{3!}$$

$$P_3(x_3+sh) = 1.101 + 0.7660625s + 0.40637 5\frac{s(s+1)}{2!} + 0.09375\frac{s(s+1)(s+2)}{3!}$$

If $x = -\frac{1}{3}$ $\quad 0+s\cdot\frac{1}{4} = -\frac{1}{3}$ $\quad s = -\frac{4}{3}$

$$P_3(-\tfrac{4}{3}) = 1.101 + 0.7660625(-\tfrac{4}{3}) + 0.40637 5\frac{(-\frac{4}{3})(-\frac{1}{3})}{2!} + 0.09375\frac{(-\frac{4}{3})(-\frac{1}{3})(\frac{2}{3})}{3!} = 0.1745185$$