MAC 2313-02 Calculus III
Final Exam
12/13/2012 Print Name ____________________

1. (10pt) The vector equation of a curve C is $\vec{r}(t) = \langle 3\cos t, 3\sin t, 4t\rangle$.
   (a) Find the arc length between the points $P(3,0,0)$ and $Q(3,0,8\pi)$ on the curve C.

   (b) Find the coordinates of a point R other than $P(3,0,0)$ on the curve C, such that the arc length between R and Q is the same as the arc length between P and Q.

2. (10pt) $f(x,y,z) = xe^y + ye^z + ze^x$.

   (a) (3pt) Find the gradient of $f(x,y,z)$.

   (b) (4pt) Find the directional derivative of f at the point $(0,0,0)$ in the direction of $\langle 0,2,1\rangle$.

   (c) (3pt) Find the maximum rate of change of f at the point $(0,0,0)$. In which direction does it occur?

3. (10pt) Find the local maximum and minimum values and saddle points of $f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10$, if they exist.

4. (10pt) Sketch the region of integral and calculate the iterated integral by first reversing the order of integration.

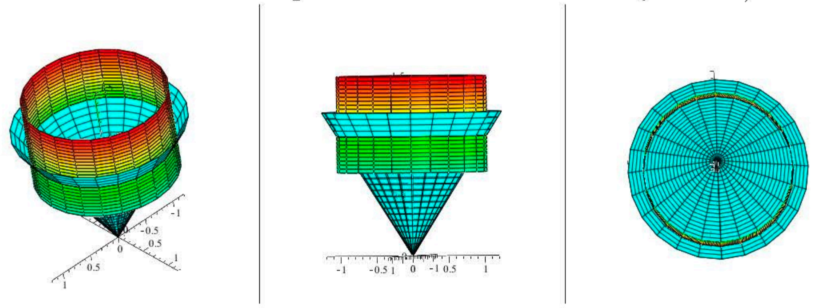
$$\int_0^3 \int_{\sqrt{y/3}}^1 e^{x^3}\, dx\, dy$$

5. (10pt) Find the volume of the solid bounded by the two paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

6. (10pt) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane and below the cone $z = \sqrt{x^2 + y^2}$.

7. (10pt) Use Stokes' theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x^2 z^2 \mathbf{i} + y^2 z^2 \mathbf{j} + xyz\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 1$, oriented upward.

   (Hint: Stokes' theorem: $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$. The boundary of S (i.e. the curve C) is the intersection of the paraboloid with the the cylinder.)



8. (10pt) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + \mathbf{k}$ and S is the part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 2$, oriented upward.

9. (10pt) Find the area of part of the surface $3x + 4y + z = 6$ that lies in the first octant.

10. (10pt) $\mathbf{F}(x, y) = x^2\mathbf{i} + y^2\mathbf{j}$.

(a) Show that $\mathbf{F}$ is conservative and find f such that $\nabla f(x, y) = \mathbf{F}(x, y)$.

(b) Use the result in part (a) and the Fundamental Theorem for line integral to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the arc of the parabola $y = 2x^2$ from $(0, 0)$ to $(1, 2)$.