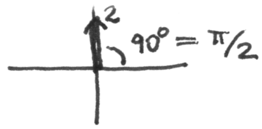
PROBLEM A.1:

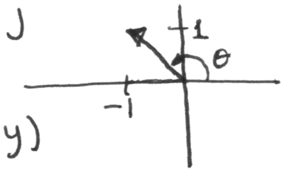
(a) $z = j2$



$z = 2e^{j\pi/2} = 2\angle 90^\circ$

(b) $z = -1 + j$

$(-1, 1) = (x, y)$

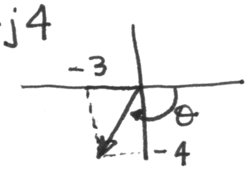


$\theta = 135^\circ = 3\pi/4$ radians

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$z = \sqrt{2}\, e^{j3\pi/4} = \sqrt{2}\, e^{j2.356}$

(c) $z = -3 - j4$



$r = \sqrt{3^2 + 4^2} = 5$ $\qquad 5\angle -126.87^\circ$

$\theta = \mathrm{Tan}^{-1}\left(\frac{-4}{-3}\right) = -126.87^\circ$

convert to radians: $-126.87\left(\frac{\pi}{180}\right) = -2.21 = -0.705\pi$

$z = 5e^{-j0.705\pi} = 5e^{-j2.21}$

(d) $z = (0, -1)$

$\theta = -90^\circ = -\pi/2$ rads.

$z = 1e^{-j\pi/2}$

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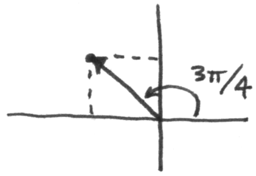
PROBLEM A.2:

(a) $z = \sqrt{2}\, e^{j3\pi/4}$

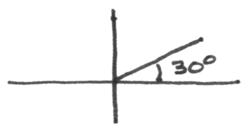


$3\pi/4 = 135^\circ$

$$z = \sqrt{2}\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right) = \sqrt{2}\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = -1 + j1$$

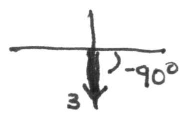
(b) $z = 1.6\angle\pi/6 = 1.6\, e^{j\pi/6} = 1.6\left(\cos\frac{\pi}{6} + j\sin\pi/6\right)$

$= 1.6\angle 30^\circ$



$= 1.6\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = 1.386 + j0.8$

(c) $z = 3e^{-j\pi/2} = 3\angle -90^\circ$



$z = -3j$

(d) $z = 7\angle 7\pi = 7\angle\pi = 7e^{j\pi} = -7 + j0$ (subtract multiple of 2π)

$= 7\angle 1260^\circ$

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PROBLEM A.3:

(a) $j^3 = j \cdot j^2 = j(-1) = -j = 0 - j$

(b) $e^{j(\pi + 2\pi m)} = e^{j\pi} e^{j2\pi m} = (-1)(1) = -1 = -1 + j0$

or,

$$= (\cos(\pi) + j\sin(\pi))(\cos(2\pi m) + j\sin(2\pi m))$$

$$= (-1 + j0)(1 + j0)$$

$$= (-1)(1) = -1 = -1 + j0$$

(c) $j^{2n} = (j^2)^n = (-1)^n = \begin{cases} +1 & \text{if } n \text{ is even} \\ -1 & \text{if } n \text{ is odd} \end{cases}$

(d) $j^{1/2} = (e^{j\pi/2})^{1/2} = e^{j\pi/4} = \boxed{\sqrt{2}/2 + j\sqrt{2}/2}$

But there is a second solution:

$$j^{1/2} = (e^{j\pi/2} e^{j2\pi})^{1/2} = e^{j\pi/4} e^{j\pi} = -e^{j\pi/4}$$

equals -1

$$j^{1/2} = -e^{j\pi/4} = \boxed{-\sqrt{2}/2 - j\sqrt{2}/2}$$

2 SOLUTIONS

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PROBLEM A.4:

(a)
$$3e^{j2\pi/3} = -3/2 + j\,3\sqrt{3}/2$$
$$-4e^{-j\pi/6} = -4\frac{\sqrt{3}}{2} + j\,2$$
ADD THESE $\Rightarrow -4.964 + j\,4.598$

$$\text{ANS} = 6.766\,e^{j0.762\pi} \quad (\text{ANGLE} = 137.2^\circ \text{ or } 2.394 \text{ rads})$$

(b)
$$\sqrt{2} - j2 = 2.449\,e^{-j0.304\pi} \quad (\text{ANGLE} = -54.74^\circ \text{ or } -0.955 \text{ rads})$$
$$(\sqrt{2}-j2)^8 = (2.449)^8\,e^{-j8(0.304)\pi}$$
$$= 1296\,e^{-j0.433\pi} \quad (\text{ANGLE} = -77.88^\circ \text{ or } -1.36 \text{ rads})$$

(c)
$$(\sqrt{2}-j2)^{-1} = \frac{1}{2.449}\,e^{+j0.304\pi} = 0.4082\,e^{+j0.304\pi}$$

(d)
$$(\sqrt{2}-j2)^{1/2} = \left(2.449\,e^{-j0.304\pi}\,e^{j2\pi\ell}\right)^{1/2}$$
$$= 1.565\,e^{-j0.152\pi}\,e^{j\pi\ell}$$
ℓ = integer

There are two answers:
$$1.565\,e^{-j0.152\pi} \quad \text{and} \quad 1.565\,e^{j0.848\pi}$$
$\ell = 1$

(e)
$$\Im m\{j e^{-j\pi/3}\} = \Im m\{e^{j\pi/2} e^{-j\pi/3}\}$$
$$= \Im m\{e^{j\pi/6}\}$$
$$= \sin(\pi/6) = 1/2$$

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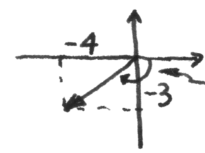
PROBLEM A.5:

(a) $z_1^* = (-4+j3)^* = -4-j3$

$= 5e^{-j0.795\pi} = 5e^{-j2.50}$



$\theta = -143.13^\circ$

(b) $z_2^2 = (1-j)^2 = 1-2j+(-j)^2 = 1-2j-1 = 0-2j = -2j$

$= (\sqrt{2}e^{-j\pi/4})^2 = 2e^{-j\pi/2} = -j2$

(c) $z_1 + z_2^* = -4+j3+(1-j)^* = -4+j3+1+j = -3+j4$

in polar: $5e^{j0.705\pi}$ (ANGLE $= 126.87^\circ = 2.214$ rads)

(d) $jz_2 = j(1-j) = j-j(j) = j+1 = 1+j$

$= e^{j\pi/2}(\sqrt{2}e^{-j\pi/4}) = \sqrt{2}e^{+j\pi/4}$

(e) $z_1^{-1} = 1/z_1 = 1/(-4+j3) = (-4-j3)/(4^2+3^2) = \frac{1}{25}(-4-j3)$

use part (a) $z_1^{-1} = \frac{1}{5}e^{-j0.795\pi}$ ($-143.13^\circ = -2.50$ rads)

(f) $z_1/z_2 = z_1 z_2^*/|z_2|^2 = \dfrac{(-4+j3)(1+j)}{1^2+1^2} = \dfrac{-4-3+j3-j4}{2}$

$= -\frac{7}{2} - j\frac{1}{2} = \frac{1}{2}\sqrt{50}e^{-j0.955\pi}$ (ANGLE $= -171.8^\circ = -3$ rads)

MAG=3.54

(g) $e^{z_2} = e^{1-j} = e\,e^{-j} = e(\cos(1) - j\sin(1)) = 1.469 - j2.287$

This is polar form

Mag $= e$ angle $= -1$ rad $= -57.3^\circ = -0.318\pi$ rads

(h) $z_1 z_1^* = (-4+j3)(-4+j3)^* = (-4+j3)(-4-j3)$

$= (-4)^2 + 3^2 = 16+9 = 25$

(i) $z_1 z_2 = (-4+j3)(1-j) = -4+j3+j4+3 = -1+j7$

(polar) $= \sqrt{50}\,e^{j0.545\pi} = 7.07e^{j1.713}$ (98.13°)

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PROBLEM A.6:

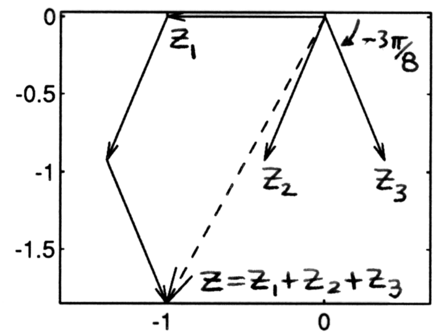
Simplify the following complex-valued sum: $z = e^{j9\pi/3} + e^{-j5\pi/8} + e^{j13\pi/8}$

$$z_1 = e^{j9\pi/3} = e^{j3\pi} = -1 + j0$$

$$z_2 = e^{-j5\pi/8} = -.383 - j.924$$

$$z_3 = e^{j13\pi/8} = e^{-j3\pi/8} = .383 - j.924$$

$$z = -1 - j1.848 = 2.1e^{-j2.07}$$



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Z = X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
-1	3.674e-16	1	3.142	1.000	180.00
-0.3827	-0.9239	1	-1.963	-0.625	-112.50
0.3827	-0.9239	1	-1.178	-0.375	-67.50
-1	-1.848	2.101	-2.067	-0.658	-118.42

PROBLEM A.7:

(a) $z = -3 + j4 = 5e^{j0.705\pi}$ (ANGLE $= 126.87°$ or 2.214 rads)

$$\frac{1}{z} = \frac{1}{5}e^{-j0.705\pi}$$

(b) $z = -2 + j2 = 2\sqrt{2}\,e^{j3\pi/4}$

SUBTRACT 4π

$$z^5 = (2\sqrt{2})^5 e^{j15\pi/4} = 128\sqrt{2}\;e^{-j\pi/4}$$

(c) $z = -5 + j13$

$$|z|^2 = z z^* = (-5+j13)(-5-j13)$$

$$= 25 + 169 = 194$$

(d) $\Re e\{(-2+j5)e^{-j\pi/2}\}$

EQUALS $-j$

$$= \Re e\{(-2+j5)(-j)\}$$

$$= \Re e\{2j + 5\} = 5$$

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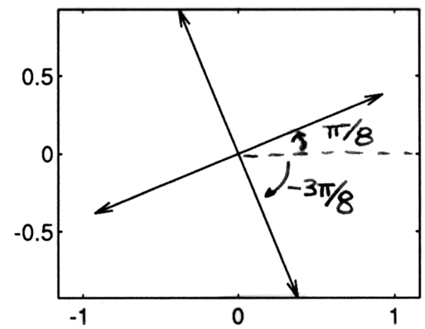
PROBLEM A.8:

Solve the following equation for z: $z^4 = j$

$$z^4 = j = e^{j\pi/2} = e^{j\pi/2}e^{j2\pi\ell}$$

$$z = \left(e^{j\pi/2}e^{j2\pi\ell}\right)^{1/4}\;;\quad \ell = 0,1,2,3$$

$$z = \left\{e^{j\pi/8},\; e^{j5\pi/8},\; e^{j9\pi/8},\; e^{j13\pi/8}\right\}$$



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Z = X	+ jY	Magnitude	Phase	Ph/pi	Ph(deg)
0.9239	0.3827	1	0.393	0.125	22.50
-0.3827	0.9239	1	1.963	0.625	112.50
-0.9239	-0.3827	1	-2.749	-0.875	-157.50
0.3827	-0.9239	1	-1.178	-0.375	-67.50

PROBLEM A.9:

$$z_0 = e^{j2\pi/N} \quad \Rightarrow \quad (z_0)^{N-1} = \left(e^{j2\pi/N}\right)^{N-1}$$

$$z_0^{N-1} = e^{j2\pi(N-1)/N} = e^{j2\pi\left(\frac{N}{N}-\frac{1}{N}\right)} = e^{j2\pi} e^{-j2\pi/N}$$

But $e^{j2\pi} = \cos(2\pi) + j\sin(2\pi) = 1 + j0 = 1$

$$\therefore \quad z_0^{N-1} = e^{-j2\pi/N} = \left(e^{j2\pi/N}\right)^{-1} = z_0^{-1} = 1/z_0$$

PROBLEM A.10:

$$-j = 1e^{-j\pi/2}$$

But we can also multiply by $e^{j2\pi\ell}$ with ℓ = integer

$$-j = 1e^{-j\pi/2}e^{j2\pi\ell}$$

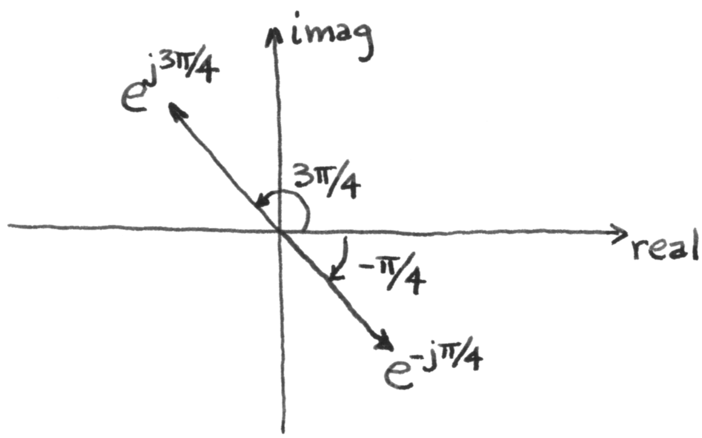
$$(-j)^{1/2} = \left(1e^{-j\pi/2}e^{j2\pi\ell}\right)^{1/2} = 1e^{-j\pi/4}e^{j\pi\ell}$$

When ℓ = integer, $e^{j\pi\ell}$ is either

$$\begin{cases} e^{j\pi\ell} = 1 & \text{when } \ell \text{ is even.} \\ e^{j\pi\ell} = e^{j\pi} = -1 & \text{when } \ell \text{ is odd.} \end{cases}$$

Thus

$$(-j)^{1/2} = \begin{cases} e^{-j\pi/4} & \text{for } \ell \text{ even} \\ e^{-j\pi/4}e^{j\pi} = e^{j3\pi/4} & \text{when } \ell \text{ is odd} \end{cases}$$



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