

Ex. Given the table of x_i and $f(x_i)$, compute the forward differences to order 4. Find $f(0.73)$ from a cubic interpolating polynomial.

x_i	$f(x_i)$	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
0	0				
0.2	0.203	0.203			
0.4	0.423	0.220	0.017		
0.6	0.684	0.261	0.041	0.024	
0.8	1.030	0.346	0.085	0.044	0.020
1.0	1.557	0.527	0.181	0.096	0.052
1.2	2.572	1.015	0.488	0.307	0.211

Since 0.73 falls between 0.6 and 0.8 and the closest 4 entries are

$$\begin{array}{cccc} 0.4 & 0.6 & 0.8 & 1 \\ x_0 & x_1 & x_2 & x_3 \end{array}$$

$$P_3(s) = 0.423 + 0.261s + 0.085 \frac{s(s-1)}{2} + 0.096 \frac{s(s-1)(s-2)}{6}$$

Since $x = 0.73$, $s = \frac{0.73 - 0.4}{0.2} = \frac{0.33}{0.2} = 1.65$

$$P_3(1.65) = 0.423 + (0.261)(1.65) + 0.085 \frac{(1.65)(0.65)}{2} + 0.096 \frac{(1.65)(0.65)(-0.35)}{6} =$$