$$g'(x) = \frac{-e^x + 1}{3} < 0 \quad \text{in } [0,1]$$

thus $g(x)$ - decreasing

$$0 < \frac{3-e}{3} = \frac{2-e+1}{3} = g(1) \le g(x) \le g(0) = \frac{2-e^0}{3} = \frac{1}{3} < 1$$

Next, we want

$$|g'(x)| \le k < 1$$

$$|g'(x)| = \frac{e^x - 1}{3} \le \frac{e-1}{3} < \frac{e}{3} < 1$$

(c) What is the rate of convergence of the fixed point iteration in part (b)?

$$O\left(\left(\frac{e-1}{3}\right)^n\right)$$

(d) For $p_0 = \frac{1}{2}$ use the formula

$$|p_n - p| \le k^n \max\{p_0 - a, b - p_0\}$$

to find the smallest number of iterations needed to obtain the solution within 10^{-4}. Take $k = \frac{2}{3}$